Background to Rock Sizing Equations

WATERWAY AND STORMWATER MANAGEMENT

Photo 1 – Rock-lined grade control structure

Photo 2 – Rock stabilisation of a river bank

1. Introduction

The purpose of fact sheet is to:

- outline the development of new procedure for the sizing of rock used in steep rock chutes;
- summarise the finding of a literature search into existing rock sizing equations that are based on either an allowable shear stress or an allowable flow velocity;
- resolve the apparent conflict between the various reported equations for sizing rock on the beds and banks of low gradient channels and steep chutes;
- assist in the determination of the best rock sizing equation for various types of stormwater and waterway structures.

The catalyst for the development of a new rock sizing equation specifically suited to steep rock spillways was the development of the IECA (Australasia) 2008 *Best Practice Erosion and Sediment Control* publication.

As a result of this research and literature review, new rock sizing procedures have been developed for sizing rock used in the construction of:

- watercourse and gully chutes (grade control structures)
- waterway riffles and fishway ramps
- artificial culvert roughness (used in the development of suitable fish passage conditions)
- rock stabilised watercourse and drainage channels
- rock-lined batter chutes
- small dam spillways
- scour protection immediately downstream of energy dissipaters
- chute, culvert and stormwater outlet structures

2. Summary of literature search outcomes

2.1 Application of shear-stress based rock-sizing equations for rock on the bed of low-gradient channels

Maynord, Ruff & Abt (1989) reported that investigations by Meyer-Peter & Muller (1948), Blench (1966), Neill (1967) and Bogardi (1978) have shown *"that the Shield's coefficient is not constant as used in many riprap design procedures, but varies directly with relative roughness (particle size/depth)".* The various reported values of the critical Shield's parameter presented in Table 1 appear to support this conclusion.

Table 1 – Summary of critical Shield's parameter

Author	F* based on use of d_{50}	F* based on use of d_{75}
Meyer-Peter & Muller [1]	0.047	0.031 [3]
Meyer-Peter & Muller [2]	0.071 [3]	0.047
E.W. Lane	0.075 [3, 4]	0.050 [4]
Shields	0.063 [4]	0.042 [3, 4]
Newbury & Gaboury	0.063 [4]	0.042 [3, 4]
McLauchlin Engineers	0.091	0.061 [3]
Chow & Henderson	0.072 [3, 4]	0.048 [4]

[1] Equations as presented in Standing Committee on Rivers and Catchments (1991).

[2] Meyer-Peter & Muller's Shield's parameter if assumed to be based on d_{75} rather than d_{50}.

[3] Based on $d_{75} = 1.5(d_{50})$.

[4] Based on specific gravity = 2.6, gravity = 9.8 m/s^2, and water density = 1000 kg/m^3.

In laminar flow conditions, shear stress consists of a force-transfer between individual moving layers of water as a result of water viscosity and the velocity gradient. There is very little flow transfer between the layers of water because agitation of the fluid particles exists only at a molecular level. In riprap design, laminar flow conditions would only exist deep within the riprap layer as part of inter-void flow. In turbulent flow, there is both energy and mass transfer between the various 'layers' (or levels) of water.

The concept of shear stress within turbulent flow may be treated as a simple energy or force transfer between water at different elevations so long as the depth of flow is significantly greater than the size of the roughness units, say, $y/d_{50} > 10$. However, in circumstances where the depth of flow approaches the size of the roughness units, then a highly irregular water surface develops and the flow takes on the characteristics commonly referred to as 'whitewater'.

In such conditions, the surface rocks are subjected to highly variable flow conditions, with macro-size jets randomly impacting upon the leading surface of exposed rocks.

In whitewater conditions, the exposed rocks are subjected more to the displacing forces of tumbling jets of water, than to what is traditionally referred to as shear stress. An effective shear stress parameter can still be calculated from the test results, but these values are much higher than those experienced under deepwater conditions.

Also, under whitewater conditions, the effects of air entrainment reduce the effective shear stress as previously discussed (refer to Section 2.12).

Under **non-uniform flow conditions on a low-gradient channel**, Equation 1 is considered the most appropriate, shear-stress based, rock-sizing equation:

(Non-uniform, low gradient flow)

$$d_{50} = \frac{SF.R.S_e}{F^*(s_r - 1)} \tag{1}$$

where:

- d_{50} = the 'design' mean rock size based on application of a suitable safety factor to the critical rock size [m]
- SF = factor of safety
- R = hydraulic radius (R = A/P) [m]
- S_e = slope of energy line [m/m]
- F* = Shield's parameter (a variable dependent on Reynolds number and rock size)
- s_r = specific gravity of rock

The critical shear stress decreases with increasing slope due to the effects of the increasing instability of a sloping rock face as the slope approaches the natural angle of repose of dumped rock. However, if the theoretical effects of embankment slope are considered separate to the shear stress, then the effective critical shear stress actually increases with increasing embankment slope due to the increasing stabilising effects of interlocking between individual rocks with increasing slope, and the effects of air entrainment.

If the theoretical effects of embankment slope are included in the shear stress equation, then the most widely adopted rock-sizing equations usually take the form of Equation 2.

(Non-uniform, steep gradient flow) $$d_{50} = \frac{R.S_e}{F*(s_r - 1)\left(\frac{1}{SF} - \frac{Tan(\theta)}{Tan(\phi)}\right)} \quad (2)$$

where:

θ = slope of channel bed [degrees]

Φ = angle of repose of rock [degrees]

In Equation 2, the Shield's parameter (F*) is not considered a constant, but increases with increasing embankment slope.

2.2 Velocity-based rock-sizing equations developed for the sizing of rock placed on the bed of low-gradient channels

Isbash (1936) investigated the stability of rounded rock dumped into flowing rivers for dam construction. His work focused on the sizing individual stones located on the downstream dam slope to resist displacement due to overtopping flow and percolation through the embankment. The results of Isbash are presented in McLaughlin Water Engineers (1986) and below as Equation 3.

$$d_{50} = \frac{V^2}{2.g.K^2.(s_r - 1)} \quad (3)$$

where:

K = 0.86 for high turbulent river flow and K = 1.2 for low turbulent river flow

The California Department of Public Works (1960) presented the Equation 4 for the sizing of rock beaching in deepwater, low-turbulent river flows.

$$W = \frac{1132 x 10^{-2} V^6 s_r}{(s_r - 1)^3 . \sin^3(70^o - \alpha)} \quad (4)$$

where:

W = weight of d_{33} rock of which 67% of rock is larger [kg]

V = average flow velocity [m/s]

s_r = specific gravity of rock

70^o = constant for randomly placed rock

α = bank slope relative to the channel bed [degrees]

This publication based it's analysis on a coefficient of uniformity for the rock ($C_u = d_{60}/d_{10}$) of 1.10. If spherical rocks are assumed, then the d_{33} rock size may be determined from Equation 5.

$$d_{33} = \frac{0.0279 . V^2}{(s_r - 1).\sin(70^o - \alpha)} \quad (5)$$

where:

d_{33} = nominal rock diameter of which 33% of the rocks are smaller [m]

Table 2 presents the equivalent rock-sizing equations for low-gradient channel beds under medium turbulence if the rock size distribution $d_{33}/d_{50} = 0.6$ is assumed. The table also presents the equivalent equation constant 'K' for the Isbash equation (Equation 3).

Table 2 – Rock size (m) for low-gradient channel bed based on California DPW

Rock type	Specific gravity	Rock sizing equations ($d_{33}/d_{50} = 0.6$)		Equ. Isbash 'K'
Sandstone	2.2	$d_{33} = 0.025\ V^2$	$d_{50} = 0.041\ V^2$	1.018
Sandstone	2.4	$d_{33} = 0.021\ V^2$	$d_{50} = 0.035\ V^2$	1.020
Granite	2.6	$d_{33} = 0.019\ V^2$	$d_{50} = 0.031\ V^2$	1.014
Basalt	2.9	$d_{33} = 0.016\ V^2$	$d_{50} = 0.026\ V^2$	1.016

Searcy (1967) developed a design chart for sizing the riprap protection of waterway banks. His results can be represented by Equation 6. This equation would appear (relative to other equations) to be suitable only for high-density rock in low turbulent flow conditions.

$$d_{50} = 0.022\,V^2 \tag{6}$$

Peterka (1984) presented Equation 7 for the sizing of riprap **downstream** of stilling basins.

$$d_{50} = 0.04\,V^2 \tag{7}$$

The wide variation in reported velocity-based rock sizing equations is most likely the result of the wide variation in flow conditions investigated by the various authors.

Most of the above equations take the form of equations 8 or 9. Typical values of the constants 'SF.A' and 'B' reported in the literature for use in Equations 8 and 9 are presented in Table 3.

Low gradient flow:

$$d_{50} = \frac{(SF.A).V^B}{(s_r - 1)} \tag{8}$$

Steep gradient flow:

$$d_{50} = \frac{A.V^B}{(s_r - 1)\left(\frac{1}{SF} - \frac{Tan(\theta)}{Tan(\phi)}\right)} \tag{9}$$

Table 3 – Typical values of equation constants 'SF.A' and 'B'

Flow conditions	SF.A	B
Deepwater, low turbulent (straight river reach)	0.035	2.00
Deepwater, highly turbulent flow (low gradient rock chutes)	0.069	2.00
Immediately downstream of hydraulic jumps and energy dissipaters	0.081	2.26

2.3 Inclusion of flow depth into velocity-based rock-sizing equations for the sizing of rock placed on the bed of low-gradient channels

Maynord (1978) presented the following equation, which included flow depth as a variable. This equation has a number of similarities with the various shear stress equations.

$$d_{50} = \frac{0.14\,(SF)}{g^{1.15}}\left(\frac{V^{2.3}}{y^{0.15}}\right) \tag{10}$$

where:

- g = acceleration due to gravity [m/s^2]
- y = depth of flow to bed at a given location [m]

If it can be assumed that Maynord's results relate to rock with a specific gravity of 2.6, then Equation 10 can be converted to:

$$d_{50} = \frac{0.018\,(SF)}{(s_r - 1)^{1.25}}\left(\frac{V^{2.3}}{y^{0.15}}\right) \tag{11}$$

Utilising dimensional analysis, Maynord, Ruff & Abt (1989) presented Equation 12 for rock placed on channel beds:

$$\frac{d}{y} = constant\left[\left(\frac{1}{(S_r - 1)}\right)^{0.5}\frac{V}{\sqrt{gy}}\right]^{2.5} \tag{12}$$

Equation 12 can be re-arranged as:

$$d_{50} = \frac{constant.(SF)}{(s_r - 1)^{1.25}}\left(\frac{V^{2.5}}{y^{0.25}}\right) \tag{13}$$

Finally, Maynord (1988) presented the above equations in the following form:

$$d_{50} = \frac{0.031(SF)}{(s_r - 1)^{1.25}} \left(\frac{V^{2.5}}{y^{0.25}} \right) \quad (14)$$

The above equations indicate that a flow depth term 'y' may be required within velocity-based equations in order to allow the results to be applicable over a wide range of flow conditions. Therefore, it would be expected that velocity-based rock sizing equations are likely to take the form of Equations 15 and 16, instead of Equations 8 and 9.

Low gradient flow:

$$d_{50} = \frac{SF.A.V^B}{y^C (s_r - 1)} \quad (15)$$

Steep gradient flow:

$$d_{50} = \frac{A.V^B}{y^C (s_r - 1)\left(\frac{1}{SF} - \frac{Tan(\theta)}{Tan(\phi)} \right)} \quad (16)$$

The work of Peirson et al. (2008) suggests that there may also be the need to incorporate an additional bed slope term into these equations to address the effects of air entrainment when the equations are applied to steep slopes.

2.4 Velocity-based rock-sizing equations developed for the sizing of rock placed on channel banks

Maynord, Ruff & Abt (1989) recommended that the same equation coefficient used for sizing rock on the channel bed, should also be used for sizing rock placed on bank slopes equal to or less than 1:2 (V:H), but a 25% increase in rock size should be applied for bank slopes of 1:1.5.

Maynord, Ruff & Abt (1989)

$$d_{50} = \frac{0.031(SF)}{(s_r - 1)^{1.25}} \left(\frac{V^{2.5}}{y^{0.25}} \right) \quad (17)$$

It should be noted that Maynord, Ruff & Abt also recommend the use of local average velocity rather than the average channel velocity.

For channel bends, the California Division of Highways (1970) presented the following velocity adjustment:

$$V_{bend} = (4/3)\ V_{avg} \quad (18)$$

However, for natural channels, Maynord (1988) recommends:

$$V_{bend} = 1.5\ V_{avg} \quad (19)$$

2.5 Design equations for the sizing of bed rock on steep gradient channels and chutes

The results of Isbash (1936) are presented in McLaughlin Water Engineers (1986) and reproduced below as Equation 20.

$$d_{50} = \frac{V^2}{2.g.K^2.(s_r - 1)} \quad (20)$$

Stephenson (1979) suggested Equation 21 as an alternative:

$$d_{50} = \left[\frac{q(\tan\theta)^{7/6}\, n_p^{\ 1/6}}{C\, g^{1/2}\, [(1 - n_p)(s_r - 1)(\cos\theta)(\tan\phi - \tan\theta)]^{5/3}} \right]^{2/3} \quad (21)$$

where:

- q = flow per unit width down the embankment [m^3/s/m]
- n_p = porosity of rock embankment
- C = coefficient (0.22 for gravel and pebbles, 0.27 for crushed stone)

Abt and Johnson (1991) reported that *"angular stone tends to interlock or wedge and subsequently resist sliding and rolling. In addition, fewer fines are required to fill the voids of crushed material compared with a similarly graded rounded stone."*

Abt and Johnson presented Equation 22 for the sizing of rock on the downstream face of rock embankments subjected to overtopping flows:

$$d_{50} = 0.595\,(SF)\,S_o^{\,0.43}\,q^{\,0.56} \tag{22}$$

McLaughlin Water Engineers (1986) presented Equation 23 for the design of rock chutes. This equation format has the benefit of including a safety factor directly linked to the angle of the bed slope relative to the natural angle of repose of the dumped rock.

$$d_{50} = \frac{y.\sin(\varphi)}{F^*.\cos(\theta).(s_r - 1).\left(\dfrac{1}{SF_F} - \dfrac{\tan(\theta)}{\tan(\phi)}\right)} \tag{23}$$

2.6 Use of average flow velocity vs local flow velocity

Rock displacement occurs as a result of local forces, local shear stresses, and local flow velocities. Therefore, it would not appear reasonable to base the rock size on flow velocities averaged over the full cross-section. Similarly, it would not appear reasonable to base rock size on the hydraulic radius of the overall channel cross-section, as this is representative of the 2-dimensional aspects of the channel section.

It would appear reasonable to base rock size on an assumed 1-dimensional flow pattern that exists above an individual rock that considers unit flow rate (q) instead of total flow (Q); local flow depth (y) instead of hydraulic radius (R); and local depth-average velocity instead of average cross-sectional velocity.

2.7 Assessment of flow velocity on the outside bank of channel bends

Rock placed on the outside bank of **major** drainage and waterway channels may be sized using the same equations provided for sizing bed rock provided the bank slope does not exceed 1:2 (V:H). A 25% increase in rock size should be applied for bank slopes of 1:1.5 (V:H). The terms used in the various equations should be redefined as follows:

- V = local flow velocity adjacent the bank as determined from Equations 24 or 25 as appropriate
- y = depth of flow measured to the toe of the bank
- S_o = slope of main channel bed (not the slope of the bank)

For **constructed trapezoidal channel bends**, the local flow velocity should be taken as 4/3 the average channel velocity, i.e.

$$V_{bend} = 1.33\,V_{avg} \tag{24}$$

For **natural channels**, the local flow velocity should be taken as 1.5 times the average channel velocity, i.e.

$$V_{bend} = 1.5\,V_{avg} \tag{25}$$

The typical extent of rock protection on channel bends is presented in Figure 1.

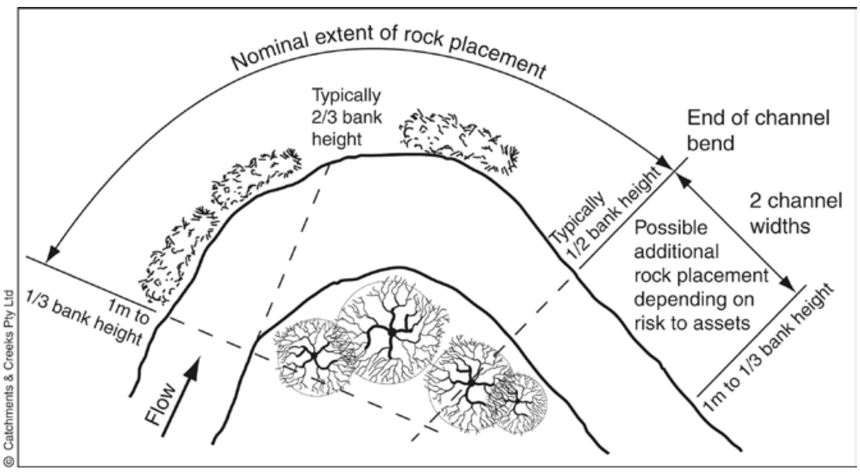


Figure 1 – Typical extent of rock protection on channel bends

2.8 Factor of safety

Various authors have recommended various safety factors and various ways of incorporating a factor of safety into the sizing of rock as used in hydraulic structures.

In some cases a safety factor (SF) is applied directly to the rock size such that:

$$d_X = SF.\ d_{X,critical} \quad (26)$$

where:

d_X = design nominal rock diameter of which X% of the rocks are smaller [m]

$d_{X,critical}$ = critical rock diameter for which movement is just expected [m]

A factor of safety (SF) applied directly to the rock size may be compared to an equivalent factor of safety applied to the design flow velocity (SF_V), or rock weight (SF_W), using the following equations.

$$SF_V = SF^{1/2} \quad (27)$$

$$SF_W = SF^{3} \quad (28)$$

The above equations are based on the following assumptions:

$$d_{50} = \text{constant} (V^2) \quad (29)$$

$$\text{Rock weight} (W_s) = (\text{volume of rock}) . g . \rho_w . s_r \quad (30)$$

$$\text{Volume of rock} = \text{constant} (d^3) \quad (31)$$

A comparison between the various types of safety factors is presented in Table 4.

Table 4 – Comparison of various safety factors

	Safety factor relative to rock size (SF)			
	1.20	**1.50**	**1.80**	**2.0**
Equivalent safety factor relative to flow velocity	1.10	1.22	1.34	1.41
Equivalent safety factor relative to weight of rock	1.73	3.88	5.83	8.00

A literature review indicates that the failure of loose rock chutes commonly relates to construction issues and the failure to utilise the specified rock size. It would also appear to be unrealistic for designers to assume that rock can always be supplied to the exact sizes specified within their design. Therefore, it would appear reasonable that a safety factor (relative to rock size) greater than 1.2 should be specified if there is expected to be poor construction supervision. As a minimum, a safety factor of 1.5 should be applied to steep chutes unless the designer knows with certainty that a specific rock size and size distribution can be supplied.

2.9 Manning's roughness of riprap

A significant number of the existing rock-sizing equations have been developed utilising the Strickler equation to obtain the Manning's roughness of the tested/monitored rock-lined surfaces. Unfortunately, this equation is only considered appropriate for deepwater flow conditions, and not for the shallow water conditions encountered by some researches. Thus care must be taken when interpreting any equations and shallow-water test results that are based on the use of the Strickler equation.

The Strickler equation for fluid motion (Equation 32) is similar to Manning's equation, except a coefficient (k) is used instead of Manning's roughness coefficient.

$$V = k\, R^{2/3}\, S^{1/2} \quad (32)$$

where:
- V = mean flow velocity [m/s]
- k = roughness coefficient (Strickler's coefficient)
- R = hydraulic radius [m]
- S = bed slope (uniform flow) [m/m]

The roughness equation developed by Strickler is presented below (Equation 33).

$$k = 21.1/(d_{50}^{1/6}) \qquad (33)$$

Strickler's work allowed the development of an equivalent Manning's roughness formula.

(metric form) $$n_o = ((d_{50})^{1/6})/21.1 \qquad (34)$$

where: n_o = Manning's roughness coefficient (assumed to be deepwater condition)

d_{50} = mean rock size for which 50% of rocks are smaller [m]

Strickler's work is often confused with formula presented by Meyer-Peter & Muller (Eqn 35).

(metric form) $$n_o = ((d_{90})^{1/6})/26.0 \qquad (35)$$

Analysing stream gauging data from New Zealand (Witheridge 2002) Equation 36 was developed for the Manning's roughness coefficient (n) for rock-lined channels in deep and shallow water. In deepwater conditions this equation is comparable with the Meyer-Peter & Muller equation.

(metric from) $$n = \frac{(d_{90})^{1/6}}{26(1 - 0.3593^{(x)^{0.7}})} \qquad (36)$$

where: X = $(R/d_{90})(d_{50}/d_{90})$

d_{90} = rock size for which 90% of rocks are smaller [m]

For 'natural' rock extracted from streambeds the relative roughness value (d_{50}/d_{90}) is typically in the range 0.2 to 0.5. For quarried rock the ratio is more likely to be in the range 0.5 to 0.8. Table 5 contains the data range for those data point where the calculated Manning's n value (using Equation 36) falls within "10% of the observed Manning's n value.

Table 5 – Data range used in determination of Equation 36

Criteria	d_{50} (mm)	d_{90} (mm)	R/d_{50}	R/d_{90}	n_o/n	d_{50}/d_{90}
Min (+/-10%)	16	90	2.31	0.73	0.284	0.080
Max (+/-10%)	112	350	55.6	12.0	1.080	0.661
Min (all data)	16	90	1.17	0.31	0.097	0.080
Max (all data)	397	1080	66.9	12.9	1.120	0.661

The rock sizing equations presented within this series of fact sheets are all based on the adoption of Manning's roughness using Equation 36.

2.10 Effects of rock shape on rock stability

Stephenson (1979) showed that due to the added interlocking of crushed rock, the required rock size (prior to threshold flow or incipient stone movement) is 87% of that required for natural rounded rock (e.g. river stone).

Abt and Johnson (1991) also reported that for rounded rock, failure occurred at flow rates 32 to 45% lower than that for angular rock at a 10% embankment slope. These observations would result in an approximate 40% increase in the required rock size when using rounded rock.

2.11 Age of rock placement

The failure of loose rock structures is contributed to by the fluctuating hydraulic pressures that exist under the rocks as a result of water passing under and around individual rocks. Observations by the author indicate that the majority of rock chute failures are associated with newly constructed chutes. In such chutes the voids are likely to be open, thus allowing free movement of water.

Once these voids become blocked with sediment, and this sediment is stabilised with vegetation (e.g. ground covers), the stability of individual rocks and thus the rock chute as a whole appears to increase.

Thus for designs based on a design storm/discharge frequency significantly greater than say 1 in 10 years, it is considered unlikely that the structure will be subjected to a near-design discharge before the structure gains the added stabilising benefits of sedimentation and vegetation cover. However, for short-term structures, such as rock-lined spillways associated with construction site sediment basins, which are often design for storm frequencies less than 1 in 10 years, the risk of a near-design discharge occurring while the structure still has open voids is relatively high. Consequently, a greater factor of safety (e.g. SF = 1.5) is recommended when designing structures that are based on a design storm less than 1 in 10 years.

2.12 Effects of air entrainment on the design of steep gradient structures

Peirson et al. (2008) reported on physical model studies carried out on steep, rock-lined spillways operating in partial whitewater conditions where air entrainment within the overtopping flow becomes an important component of the destabilising forces acting on the rocks.

Under these conditions the effective flow density decreases below that of ambient water due to air entrainment. As a result, the effective flow depth increases and the forces exerted on the rocks by the fluid decreases. In addition, rock stability is increased due to the reduced effect of buoyancy on submerged rock.

The results presented by Peirson et al. appear to demonstrate that traditional shear stress based rock sizing equations significantly overestimate rock size for steep gradient chutes and spillways operating under whitewater conditions.

2.13 Effects of interflow

Peirson et al. (2008) discussed the importance of the interflow component relative to the total overtopping flow—interflow being that portion of the total flow that moves within the rock armour. Thus the total overtopping flow is defined as the overflow plus interflow. This interflow region is typically defined from the upper surface of the underlying filter layer up to an elevation $d_{50}/3$ below the upper surface of the armour rock face.

Interflow decreases with decreasing porosity and depth of the armour layer. Porosity decreases with increasing coefficient of uniformity ($C_u = d_{60}/d_{10}$) as the rock moves from poorly graded to well graded rock. Porosity also decreases with increasing sedimentation of the air voids. Witter & Abt (1990) report that poorly graded rock (Cu approximately 1.1) had a critical discharge 8% greater than well-graded rock (Cu approximately 2.2).

2.14 Effects of bank slope

Rock placed on the outside bank of major drainage and waterway channels may be sized using the normal bed rock equation provided the bank slope does not exceed 1:2 (V:H). A 25% increase in rock size is required for bank slopes of 1:1.5 (V:H). Steeper bank slopes are complicated by the difference between the reported angle of repose and the actual critical bank slope—it is noted that the standard test procedure is influenced by the momentum introduced to the rock as a result of the dumping action.

2.15 Assessment of complex bank slopes

In cases where the channel or chute bed slope is steep (say S_o > 10%), the effective slope of the banks relative to the horizontal can be significantly greater than the bank slope relative to the bed horizontal (i.e. the bank slope shown within a given channel/chute cross-section). In such cases it is important to check that the 'effective' gradient of the bank does not exceed the maximum allowable bank slope, usually based on the natural angle of repose of the rock.

The determination of the effective bank slope (β) of complex, double-gradient slopes, can be obtained from Equation 37.

$$\tan^2(\beta) = \tan^2(\alpha) + \tan^2(\theta) \qquad (37)$$

where:

- β = bank slope relative to the horizontal [degrees]
- α = bank slope relative to the channel bed [degrees]
- θ = slope of channel bed [degrees]

2.16 Useful properties of rock

The angle of repose of rock is used in a number of rock-sizing equations presented for the design of steep-gradient chutes and spillways.

Chow (1959) suggested angles of repose for rock as per Table 6.

Table 6 – Angle of repose for rock (100–500 mm)

Rock condition	Angle of repose, Φ (degrees)
Very angular rock	41^o
Slightly angular rock	40^o
Moderately rounded rock	39^o
Rounded rock	$37–38^o$

Henderson (1966) (after Simons & Albertson, 1960) suggested angles of repose for rock as per Table 7.

Table 7 – Angle of repose for large rock (> 500 mm)

Rock condition	Angle of repose, Φ (degrees)
Very angular rock	42^o
Very rounded rock	40^o

Rock density appears to be a major factor in assessing its stability while subject to water flow. The Standing Committee on Rivers and Catchments, Victoria (1991) suggests the following rock densities:

Table 8 – Relative density (specific gravity) of rock

Rock type	Relative density (s_r)
Sandstone	2.1 to 2.4
Granite	2.5 to 3.1 (typically 2.6)
Limestone	2.6
Basalt	2.7 to 3.2

3. Overview of revised rock sizing equations

Table 9 provides the recommended design equations for sizing rock on the bed of channels and chutes. The same equations can be used for sizing rock on bank slopes equal to or less than 1:2 (V:H), but a 25% increase in rock size should be applied for bank slopes of 1:1.5.

Table 9 – Recommended rock sizing equations for channels and chutes [1]

Bed slope (%)	Design equations
Preferred equation: $S_o < 50\%$	Uniform flow conditions only, $S_e = S_o$ $$d_{50} = \frac{1.27.SF.K_1.K_2.S_o^{0.5}.q^{0.5}.y^{0.25}}{(s_r - 1)} \quad (38)$$
A simplified equation independent of flow depth: $S_o < 50\%$	Uniform flow conditions only, $S_e = S_o$ $$d_{50} = \frac{SF.K_1.K_2.S_o^{0.47}.q^{0.64}}{(s_r - 1)} \quad (39)$$
A simplified, velocity-based equation: $S_o < 33\%$	Uniform flow conditions only, $S_e = S_o$ $$d_{50} = \frac{SF.K_1.K_2.V^2}{(A - B.\ln(S_o)).(s_r - 1)} \quad (40)$$ For SF = 1.2: A = 3.95, B = 4.97 For SF = 1.5: A = 2.44, B = 4.60
A simplified, velocity-based equation: $S_o < 10\%$	Low gradient, uniform or non-uniform flow conditions $$d_{50} \approx \frac{K_1.V^{3.9}}{C.y^{0.95}(s_r - 1)} \quad (41)$$ C = 120 and 68 for SF = 1.2 and 1.5 respectively
Partially drowned waterway chutes: $S_o < 50\%$	Steep gradient, non-uniform flow conditions, $S_e \neq S_o$ $$d_{50} = \frac{1.27.SF.K_1.K_2.S_o^{0.5}.V^{2.5}.y^{0.75}}{V_o^{2.0}(s_r - 1)} \quad (42)$$
A simplified, velocity-based equation for non-critical conditions only: $S_o < 5\%$	Low gradient, non-uniform flow conditions $$d_{50} \approx \frac{K_1.V^2}{2.g.K^2(s_r - 1)} \quad (43)$$ K = 1.1 for low-turbulent deepwater flow, 1.0 for low-turbulent shallow water flow, and 0.86 for highly turbulent and/or supercritical flow

[1] The above equations, with the exception of Equation 39, are based on the Manning's 'n' roughness for rock-lined surfaced determined from Equation 36.

where:

- d_{50} = nominal rock size (diameter) of which 50% of the rocks are smaller [m]
- A & B = equation constants
- K = equation constant based on flow conditions
  - = 1.1 for low-turbulent deepwater flow, 1.0 for low-turbulent shallow water flow, and 0.86 for highly turbulent and/or supercritical flow (also refer to Table 10)
- K_1 = correction factor for rock shape
  - = 1.0 for angular (fractured) rock, 1.36 for rounded rock (i.e. smooth, spherical rock)
- K_2 = correction factor for rock grading (critical for sizing rock on steep chutes)
  - = 0.95 for poorly graded rock ($C_u = d_{60}/d_{10} < 1.5$), 1.05 for well graded rock ($C_u > 2.5$), otherwise $K_2 = 1.0$ ($1.5 < C_u < 2.5$)

q	=	flow per unit width down the embankment [$m^3/s/m$]
s_r	=	specific gravity of rock (e.g. sandstone 2.1–2.4; granite 2.5–3.1, typically 2.6; limestone 2.6; basalt 2.7–3.2)
S_e	=	slope of energy line [m/m]
S_o	=	bed slope = tan(θ) [m/m]
SF	=	factor of safety (refer to Table 11)
V	=	actual depth-average flow velocity at location of rock [m/s]
V_o	=	depth-average flow velocity based on **uniform** flow down a slope, S_o [m/s]
y	=	depth of flow at a given location [m]
θ	=	slope of channel bed [degrees]
Φ	=	angle of repose of rock (refer to Tables 6 & 7) [degrees]

Table 10 provides the 'K-values' that would be required in order for Equation 43 to produce the equivalent rock size determined from Equation 41 for **uniform flow** conditions. As can be seen from the table, as the channel slope increases and the flow becomes more turbulent, the required K-value decreases, which is consistent with the recommendations of Isbash (1936) from which the equation was developed.

Table 10 – Values of 'K' required in Equation 43 to achieve the same rock size as Equation 41 for uniform flow conditions

Bed slope (%)	**1.0**	**2.0**	**3.0**	**4.0**	**5.0**	**6.0**	**8.0**	**10.0**
K =	1.09	1.01	0.96	0.92	0.89	0.86	0.83	0.80
Flow conditions	Low turbulence ➔ ➔ ➔ ➔ ➔ ➔ ➔ ➔ ➔ Highly turbulent (whitewater)							

[1] Tabulated results are applicable to uniform flow conditions, and Manning's n based on Equation 36.

Table 11 – Recommended safety factor for use in determining rock size

Safety factor (SF)	**Recommended usage**	**Example site conditions**
1.2	• Low risk structures. • Failure of structure is most unlikely to cause loss of life or irreversible property damage. • Permanent rock chutes with all voids filled with soil and pocket planted.	• Embankment chutes where failure of the structure is likely to result in easily repairable soil erosion. • Permanent chutes that are likely to experience significant sedimentation and vegetation growth before experiencing the high flows. • Temporary (< 2 yrs) spillways with a design storm of 1 in 10 years of greater.
1.5	• High risk structures. • Failure of structure may cause loss of life or irreversible property damage. • Temporary structures that have a high risk of experiencing the design discharge while the voids remain open (i.e. prior to sediment settling within and stabilising the voids between individual rocks).	• Waterway chutes where failure of the chute may cause severe gully erosion and/or damage to the waterway. • Sediment basin or dam spillways located immediately up-slope of a residential area or busy roadway where an embankment failure could cause property flooding or loss of life. • Spillways and chutes designed for a storm frequency less than 1 in 10 years.

4. Literature review of existing rock sizing equations

4.1 The shear stress equation

The gravitational forces acting on a body of water are shown in Figure 2.

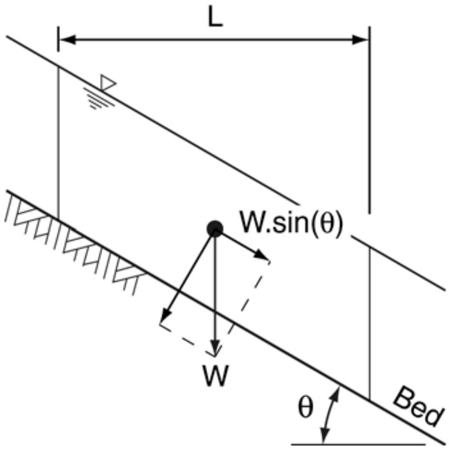


Figure 2 – Gravitational forces of body of water

For a control volume of water, the component of the weight tangential to the channel bed (F_w) is given by Equation 44.

$$F_w = W.\sin(\theta) \quad (44)$$

where:

$$W = \rho_w . g . A . L \quad (45)$$

In uniform flow conditions there is no rapid acceleration of the water, thus the weight component must be balanced by the tractive force exerted by the channel bed on the water, therefore:

$$\tau_o.P.L = W.\sin(\theta) = \rho_w.g.A.L.\sin(\theta) \quad (46)$$

where:

- F_w = weight component of water cell parallel to channel bed [N]
- W = weight of water cell [N]
- θ = slope of channel bed [degrees]
- P = wetted perimeter [m]
- ρ_w = density of water [kg/m^3]
- g = acceleration due to gravity [m/s^2]
- A = cross sectional area of channel [m^2]
- L = length of water cell [m]
- τ_o = average shear stress on the surface (bed and banks) of the channel [Pa = N/m^2]

Therefore the average shear stress may be given by:

$$\tau_o = \rho_w.g.R.\sin(\theta) \quad (47)$$

Now during uniform flow on a low-gradient channel, the energy slope has the following properties:

$$S_e = S_o = \theta = \tan(\theta) \approx \sin(\theta) \quad (48)$$

Thus:

$$\tau_o = \rho_w.g.R_h.S_o \quad (49)$$

where:

- R = hydraulic radius ($R = A/P$) [m]
- S_e = slope of energy line [m/m]
- S_o = channel bed slope [m/m]

Equation 47 represents the **average** shear stress on the bed and banks; however, the actual shear stress varies across the bed and banks. Chow (1959) indicates that the maximum shear stress on the bed and banks for a **trapezoidal** channel may be approximated by Equations 50 and 51.

Maximum shear stress on bed: $$\tau_{max} = \rho_w . g . y . \sin(\theta) \quad (50)$$

Maximum shear stress on banks: $$\tau_{max} = (0.75) . \rho_w . g . y . \sin(\theta) \quad (51)$$

where:

y = depth of flow across the channel bed [m]

The above equations are all based on uniform flow conditions. In cases where the energy slope (S_e) is significantly different from the bed slope (S_o), the water will experience either acceleration or deceleration. Under those conditions where the water is accelerating, the weight component in the direction of flow (W.sin(θ)) will be greater than the effective shear stress, i.e. there is a resulting gravitational force acting on the water causing it to accelerate. In such cases, the effective shear stress is assumed to be proportional to the energy slope rather than the bed slope.

Thus, for **non-uniform** flow on both steep and low-gradient trapezoidal channels, the following conditions apply:

Average shear stress on channel: $$\tau_o = \rho_w . g . R . \sin(\varphi) \quad (52)$$

Maximum shear stress on bed: $$\tau_{max} = \rho_w . g . y . \sin(\varphi) \quad (53)$$

Maximum shear stress on banks: $$\tau_{max} = (0.75) . \rho_w . g . y . \sin(\varphi) \quad (54)$$

where:

φ = local gradient of the energy line [degrees]

S_e = tan (φ) = slope of energy line [m/m]

and for non-uniform flow: $$\sin(\varphi) \neq \sin(\theta) \quad (55)$$

Now:

$$\sin(\varphi) = \cos(\varphi) . \tan(\varphi) = \cos(\varphi) . S_e \quad (56)$$

Thus Equations 52 to 54 may also be written as:

Average shear stress on channel: $$\tau_o = \rho_w . g . R . \cos(\varphi) . S_e \quad (57)$$

Maximum shear stress on bed: $$\tau_{max} = \rho_w . g . y . \cos(\varphi) . S_e \quad (58)$$

Maximum shear stress on banks: $$\tau_{max} = (0.75) . \rho_w . g . y . \cos(\varphi) . S_e \quad (59)$$

4.2 Rock sizing equations based on Shield’s critical shear stress

The critical shear stress on the bed of a very low gradient channel is given by Equation 60.

$$\tau_c = F^* . \rho_w . g . d_x . (s_r - 1) \quad (60)$$

where:

τ_c = critical shear stress at which point rocks begin to move [N/m^2]

F^* = Shield’s parameter (variable depending on Reynolds number and rock size, d_x)

d_x = nominal rock size (diameter) of which X% of the rocks are smaller [m]

s_r = specific gravity of rock

The following section outlines the critical shear stress Shield’s parameters presented by various authors.

4.2.1 Chow (1959) and Henderson (1966)

Critical shear stress based on d_{75} rock size:

$$\tau_c = 754.d_{75} \quad (61)$$

If the specific gravity of the rock (s_r) is assumed to be 2.6, gravity is 9.8 m/s^2, and the water density is 1000 kg/m^3, then Equation 31 becomes:

$$\tau_c = (0.048)\rho_w.g.d_{75}(s_r - 1) \quad (62)$$

4.2.2 Meyer-Peter and Muller formula (reported by the Standing Committee on Rivers and Catchments, Victoria, 1991)

For a flat bed the Meyer-Peter and Muller formula (1948) is presented as:

$$\tau_c = (0.047)\rho_w.g.d_{50}(s_r - 1) \quad (63)$$

It would initially appear (refer to Table 12) that Equation 63 would be more consistent with other reported critical shear stress equations if the rock size used in the equation was actually d_{75} or d_{90} instead of the reported d_{50}. However, this may actually reflect the variability of the critical shear stress for various flow conditions.

4.2.3 Newbury and Gaboury (1993)

Critical shear stress based on d_{50} rock size:

$$\tau_c = 980.d_{50} \quad (64)$$

If the specific gravity of the rock (s_r) is assumed to be 2.6, gravity is 9.8 m/s^2, and the water density is 1000 kg/m^3, then the above equation becomes:

$$\tau_c = (0.063)\rho_w.g.d_{50}(s_r - 1) \quad (65)$$

4.2.4 E. W. Lane (presented in L.W. Mays, 2005)

Critical shear stress developed for clear water flow based on d_{75} rock size:

$$\tau_c = 785.d_{75} \quad (66)$$

If the specific gravity of the rock (s_r) is assumed to be 2.6, gravity is 9.8 m/s^2, and the water density is 1000 kg/m^3, then the above equation becomes:

$$\tau_c = (0.050)\rho_w.g.d_{75}(s_r - 1) \quad (67)$$

4.2.5 Shields (presented in L.W. Mays, 2005)

Critical shear stress based on d_{50} rock size:

$$\tau_c = 990.d_{50} \quad (68)$$

If the specific gravity of the rock (s_r) is assumed to be 2.6, gravity is 9.8 m/s^2, and the water density is 1000 kg/m^3, then Equation 27 becomes:

$$\tau_c = (0.063)\rho_w.g.d_{50}(s_r - 1) \quad (69)$$

4.2.6 Application of the Shield's parameter and shear stress equation

From Equation 50 the **critical** rock size (d_{50}) on the **bed** of a **low-gradient channel** under **uniform flow conditions** is given by:

$$\tau_c = F^*.\rho_w.g.d_x.(s_r - 1) \quad (70)$$

incorporating Equation 49:

$$\tau_c = F^*.\rho_w.g.d_x.(s_r - 1) = \rho_w.g.R.S_o \quad (71)$$

if F* is based on d_{50}, then:

$$F^*.d_{50}.(s_r - 1) = R.S_o \quad (72)$$

rearranging produces:

$$d_{50,critical} = \frac{R.S_o}{F^*(s_r - 1)} \quad (73)$$

where:

$d_{50,critical}$ = mean rock diameter for which movement is expected to commence [m]

A safety factor (SF) can be introduced to determine the **design** rock size:

Uniform flow:
$$d_{50} = \frac{SF.R.S_o}{F^*(s_r - 1)} \quad (74)$$

A comparison can be made with the allowable velocity method (Section 4.3) where:

$$d_{50} = K.V^2 \quad (75)$$

From Manning's equation:
$$V^2 = \frac{1}{n^2} R^{4/3} S_o \quad (76)$$

thus,

$$K = \frac{SF.n^2}{F^*(s_r - 1)R^{1/3}} \quad (77)$$

where:

d_{50} = nominal rock diameter of which 50% of the rocks are smaller [m]
K = constant which varies with the degree of local turbulence
V = average flow velocity [m/s]
n = Manning's roughness value [dimensionless]

Suggested values are SF = 1.5, and F* = 0.07 (applicable when determining the d_{50} rock size – refer to Table 12).

Technical Note 1:

The above equations only apply to low-gradient channels because the potential effects of gravity contributing to the displacement of rocks down a slope have not been considered.

It should be noted that Maynord, Ruff & Abt (1989) report that investigations by Meyer-Peter & Muller 1948, Blench 1966, Neill 1967 and Bogardi 1978, have shown "that the Shield's coefficient is not constant as used in many riprap design procedures, but varies directly with relative roughness (particle size/depth)". For further discussion, refer to Section 4.3.9.

Under **non-uniform flow conditions** on a low-gradient channel, Equation 74 becomes:

Non-uniform flow:
$$d_{50} = \frac{SF.R.\cos(\varphi).S_e}{F^*(s_r - 1)} \quad (78)$$

However, for a low-gradient channel the energy slope (φ) is likely to be small, thus:

$$\cos(\varphi) \approx 1.0 \quad (79)$$

and:

$$d_{50} = \frac{SF.R.S_e}{F^*(s_r - 1)} \quad (80)$$

A summary of the various recommended Shield's parameters is provided in Table 12.

Table 12 – Summary of Shield's parameter

Author	F* for d_{50}	F* for d_{75}
Meyer-Peter & Muller [1]	0.047	0.031 [3]
Meyer-Peter & Muller [2]	0.071 [3]	0.047
E.W. Lane	0.075 [3, 4]	0.050 [4]
Shields	0.063 [4]	0.042 [3, 4]
Newbury & Gaboury	0.063 [4]	0.042 [3, 4]
McLauchlin Engineers (refer to Section 11.1)	0.091	0.061 [3]
Chow & Henderson	0.072 [3, 4]	0.048 [4]

[1] Equations as presented in Standing Committee on Rivers and Catchments (1991).

[2] Meyer-Peter & Muller's Shield's parameter if assumed to be based on d_{75} rather than d_{50}.

[3] Based on $d_{75} = 1.5(d_{50})$ refer to Table 9.

[4] Based on specific gravity = 2.6, gravity = 9.8 m/s^2, and water density = 1000 kg/m^3.

4.3 Rock sizing equations based on an allowable velocity

The allowable velocity method for sizing rock is based on the development of a direct relationship between the required rock size and the local average flow velocity. These equations usually take the form of:

$$d_{50} = A.V^B \tag{81}$$

where:

A = constant that typically varies with the degree of turbulence, rock texture (e.g. rounded or fractured) and the required factor of safety

B = constant that typically varies with the degree of turbulence

Typically, these allowable flow velocity equations are only applicable for low-gradient channels (say less than 10%) operating under deepwater conditions (i.e. $y >> d_{50}$) where there is only a minor gravity force component down the face of the slope, and whitewater flow conditions do not exist.

4.3.1 Isbash (1936)

Isbash investigated the stability of rounded rock dumped into flowing rivers for dam construction. His work focused on sizing individual stones located on the downstream dam slope to resist displacement due to overtopping flow and percolation through the embankment.

The results of Isbash (1936) (reported in McLaughlin Water Engineers, 1986) is presented below as Equation 82.

$$d_{50} = \frac{V^2}{2.g.K^2.(s_r - 1)} \tag{82}$$

where:

K = equation constant based on flow conditions

Isbash recommended K = 0.86 for high turbulent river flow and K = 1.2 for low turbulent river flow, thus:

Low turbulent channel flow: $$d_{50} = \frac{(0.035).V^2}{(s_r - 1)} \tag{83}$$

High turbulent channel flow: $$d_{50} = \frac{(0.069).V^2}{(s_r - 1)} \tag{84}$$

Results of Isbash's equation are summarised in Table 13.

Table 13 – Rock size (m) for low-gradient channel bed based on Isbash's equation

Rock type	specific gravity	Low turbulent flow	High turbulent flow
Sandstone	2.2	$d_{50} = 0.030\ V^2$	$d_{50} = 0.057\ V^2$
Sandstone	2.4	$d_{50} = 0.025\ V^2$	$d_{50} = 0.049\ V^2$
Granite & Limestone	2.6	$d_{50} = 0.022\ V^2$	$d_{50} = 0.043\ V^2$
Basalt	2.9	$d_{50} = 0.019\ V^2$	$d_{50} = 0.036\ V^2$

4.3.2 California Department of Public Works (1960)

Equation 85 was presented by California Department of Public Works (1960) for deepwater, low-turbulent river flows.

$$W = \frac{1132 \times 10^{-2}\, V^6\, s_r}{(s_r - 1)^3 . \sin^3(70^o - \alpha)} \qquad (85)$$

where:

- W = weight of d_{33} rock of which 67% of rock is larger [kg]
- V = average flow velocity [m/s]
- s_r = specific gravity of rock
- 70^o = constant for randomly placed rock
- α = bank slope relative to the channel bed [degrees]

This publication based it's analysis on a coefficient of uniformity ($C_u = d_{60}/d_{10}$) of 1.10. If spherical rocks are assumed then the nominal diameter may be determined from Equation 86.

$$d = \left(\frac{6.W}{\pi . s_r\, x10^3} \right)^{1/3} \qquad (86)$$

where:

- d = nominal rock diameter [m]

Thus the d_{33} rock size may be determined from Equation 87.

$$d_{33} = \frac{0.0279 . V^2}{(s_r - 1) . \sin(70^o - \alpha)} \qquad (87)$$

where:

- d_{33} = rock diameter of which 33% of the rocks are smaller [m]

For rock with a specific gravity (s_r) of 2.6 on a low-gradient channel bed (i.e. $\alpha = 0^o$) then:

$$d_{33} = 0.019\ V^2 \qquad (88)$$

Table 14 presents the equivalent rock-sizing equations for low-gradient channel beds under medium turbulence if the rock size distribution as presented in Table 8 (i.e. $d_{33}/d_{50} = 0.6$) and the equivalent equation constant 'K' for the Isbash equation (Eqn 82). These equations compare well with the rock sizing equations presented in Table 13.

Table 14 – Rock size (m) for low-gradient channel bed based on California Department of Public Works

Rock type	s_r	Rock sizing equations ($d_{33}/d_{50} = 0.6$)		Equ. Isbash 'K'
Sandstone	2.2	$d_{33} = 0.025\ V^2$	$d_{50} = 0.041\ V^2$	1.018
Sandstone	2.4	$d_{33} = 0.021\ V^2$	$d_{50} = 0.035\ V^2$	1.020
Granite/Limestone	2.6	$d_{33} = 0.019\ V^2$	$d_{50} = 0.031\ V^2$	1.014
Basalt	2.9	$d_{33} = 0.016\ V^2$	$d_{50} = 0.026\ V^2$	1.016

4.3.3 Searcy (1967)

Searcy (1967) developed a rock-sizing chart for sizing riprap protection of waterway banks. His design chart can be represented by Equation 89. This equation would appear (relative to other publications) to be suitable only for high-density rock in low turbulent flow conditions.

$$d_{50} = 0.022\, V^2 \qquad (89)$$

4.3.4 J.P.Bohan (1970)

For sizing rock placed immediately downstream of culvert and pipe outlets, Bohan presents the following equations:

For low tailwater ($TW<D_o/2$): $$d_{50} = 0.25\, D_o\, F_o \qquad (90)$$

Which can be converted to: $$d_{50} = 0.080\, D_o^{1/2}\, V_o \qquad (91)$$

For high tailwater ($TW>D_o/2$): $$d_{50} = 0.25\, D_o\, F_o - 0.15\, D_o \qquad (92)$$

Which can be converted to: $$d_{50} = 0.080\, D_o^{1/2}\, V_o - 0.15\, D_o \qquad (93)$$

where:

TW = Tailwater depth relative to pipe invert [m]

D_o = internal diameter of outlet pipe [m]

F_o = Froude number of flow at the pipe exit [dimensionless]

4.3.5 Maynord (1978)

Maynord (1978) presented the following equations for sizing rock on channel beds and banks up to 3:1 (H:V) under flow conditions of deceleration flow.

Test result best fit: $$d_{50} = 0.14\, (SF)(y)\, F^{2.3} \qquad (94)$$

Recommended design: $$d_{50} = 0.22\, (SF)(y)\, F^{3} \qquad (95)$$

where:

SF = Recommended safety factor, SF = 1.15 (equivalent to a safety factor based on rock weight, SF_W = 1.5)

y = depth of flow within a trapezoidal channel [m]

F = Froude number based on flow depth (y) rather than hydraulic radius [dimensionless]

$$F = \frac{V}{\sqrt{g.y}} \qquad (96)$$

The above equations can be expanded to give:

Test result best fit: $$d_{50} = \frac{0.14\,(SF)}{g^{1.15}}\left(\frac{V^{2.3}}{y^{0.15}}\right) \qquad (97)$$

Recommended design: $$d_{50} = \frac{0.22\,(SF)}{g^{1.5}}\left(\frac{V^{3}}{y^{0.5}}\right) \qquad (98)$$

Now these equations can be compared to the traditional shear stress equations:

Shear stress equation: $$d_{50} = \frac{SF.R.S_o}{F^*(s_r - 1)} \qquad (99)$$

$$d_{50} = \frac{SF.n^2}{F^*(s_r - 1)}\left(\frac{V^2}{R^{1/3}}\right) \qquad (100)$$

From the above it would appear that Maynord's original test results produced an equation that best aligns with the traditional shear stress equation. It is also noted that Maynord's analysis was performed on a trapezoidal channel where for average channel conditions may be defined by Equation 101.

(typical channel conditions) $$R = 0.6535(y) \tag{101}$$

thus, $$d_{50} = \frac{1152(SF)n^2}{F^*(s_r - 1)}\left(\frac{V^2}{y^{1/3}}\right) \tag{102}$$

An important outcome from this hydraulic study was the observation that in trapezoidal channels with side slope of 3:1 to 4:1, failure began to occur on the bed and banks at the same or similar flow conditions; however, for side slopes of 2:1 failure was always initiated on the banks. This indicates that for side slopes of 3:1 to 4:1 the combined effects of a decrease in shear stress on the banks plus interlocking forces between the rocks, is equally balanced by the increase in displacement forces resulting from gravity acting of the steep bank.

4.3.6 Bos, Replogle & Clemmens (1984)

Bos, Reploge, and Clemmens, (1984) present rock sizing equations suitable for rocks placed within the zone of highly turbulent water immediately **downstream** of the end sill of an energy dissipater (i.e. not within the main energy dissipation zone).

$$d_{40} = 0.038\ V^{2.26} \tag{103}$$

If it is assumed that $d_{40}/d_{50} = 0.75$ (see Table 9), then:

$$d_{50} = 0.050\ V^{2.26} \tag{104}$$

If it is assumed that Equation 104 is based on rock of a specific gravity (s_r) of 2.6, then:

$$d_{50} = \frac{(0.081).V^{2.26}}{(s_r - 1)} \tag{105}$$

4.3.7 Peterka (1984)

Peterka (1984) presented the following rock-sizing equation for riprap **downstream** of stilling basins:

$$d_{50} = 0.04\ V^2 \tag{106}$$

4.3.8 Maynord (1988)

Maynord (1988) presented that following rock-sizing equation for rock with a sizing gradation of $d_{85}/d_{15} = 1.8$ to 4.6:

$$\frac{d_{30}}{y} = 0.30(SF)\left[\left(\frac{1}{(S_r - 1)}\right)^{0.5}\frac{V}{\sqrt{gy}}\right]^{2.5} \tag{107}$$

If it is assuming that $d_{30}/d_{50} = 0.55$ (Table 9):

$$d_{50} = \frac{0.031(SF)}{(s_r - 1)^{1.25}}\left(\frac{V^{2.5}}{y^{0.25}}\right) \tag{108}$$

4.3.9 Maynord, Ruff & Abt (1989)

Using dimensional analysis Maynord, Ruff & Abt (1989) suggested the following rock-sizing equation for rock placed on channel beds:

$$\frac{d}{y} = \text{constant}\left[\left(\frac{1}{(S_r - 1)}\right)^{0.5}\frac{V}{\sqrt{gy}}\right]^{2.5} \tag{109}$$

4.3.10 American Society of Civil Engineers (1992)

For the sizing of rocks for outlet protection immediately downstream of culvert and pipe outlets, ASCE (1992) presents the following equations:

$$d_{50} = \frac{0.066\,(Q)^{4/3}}{TW\,(D_o)} \qquad (110)$$

For TW = D_o/2: $$d_{50} = 0.0957\, D_o^{\,2/3}\, V_o^{\,4/3} \qquad (111)$$

where:

- TW = tailwater depth relative to the outlet invert level [m]
- D_o = internal diameter of the outlet pipe or culvert [m]
- V_o = outlet velocity [m/s]

4.4 Rock sizing for the banks of low gradient (river) channels

4.4.1 Maynord, Ruff & Abt (1989)

Maynord, Ruff & Abt (1989) reported that the following three factors impacted on rock stability within channel banks:

- The gravitational component of rock weight acting down the bank slope.
- The influence of side slope on local velocity [and shear stress]. Steep side slopes often tend to move the maximum local velocity away from the toe of the slope (due to the effects of bank friction on bed velocity). Flatter side slopes allow the maximum velocity to occur over the toe of the slope. It is noted that flow velocity near the toe of the batter is important because bank rock located near the toe of the bank, can experience the combined effects of high shear stress plus the gravity component of its weight.
- The fact that rock placed on steep banks experience interlocking (due to weight of up-slope rocks), which rock on the channel bed typically does not experience. On the channel bed the rocks behave more like individual units.

Test results showed that for side slopes of 1.5:1 to 3:1 (H:V) the coefficient associated with the rock sizing equation varied only slightly and thus they concluded the “limited influence of side slope angle on riprap size”. They recommend the same equation coefficient that is used for sizing rock of the channel bed, also be used for sizing rock on bank slopes equal to or less than 2:1, but a 25% increase in rock size should be applied for bank slopes of 1.5:1. Note this does **not** apply to rock placed on the face of steep gradient chutes.

Maynord, Ruff & Abt recommended a safety factor (applicable to rock size) of SF = 1.2. The resulting design equation is presented below as Equation 112. This equation has been developed for is rock with d_{85}/d_{15} = 1.8 to 4.6, assumed to be equivalent to, d_{50}/d_{90} = 0.6 to 0.23.

$$\frac{d_{30}}{y} = 0.30\,(SF)\left[\left(\frac{1}{(S_r - 1)}\right)^{0.5} \frac{V}{\sqrt{g\,y}}\right]^{2.5} \qquad (112)$$

If it is assumed that d_{30}/d_{50} = 0.55, then:

$$d_{50} = \frac{0.031(SF)}{(s_r - 1)^{1.25}}\left(\frac{V^{2.5}}{y^{0.25}}\right) \qquad (113)$$

It should be noted that Maynord, Ruff & Abt recommend the use of local average velocity rather than the average channel velocity—the latter being influenced by channel shape. For channel bends, Maynord, Ruff & Abt referred to California Division of Highways (1970), which presented the following design equation:

Design velocity on channel bend: $$V_{bend} = (4/3)\, V_{avg} \qquad (114)$$

where:

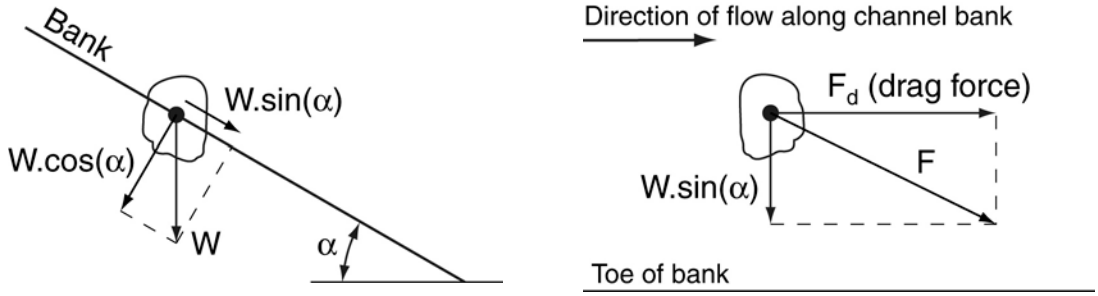
- V_{bend} = the maximum depth-average velocity in the bend over the toe of the side slope
- V_{avg} = the average channel velocity

However, for natural channels, Maynord (1988) recommends:

Design velocity on channel bend: $V_{bend} = 1.5\ V_{avg}$ (115)

4.4.2 Standing Committee on Rivers and Catchments, Victoria (1991)

The drag forces of a rock are shown in Figure 3.



(a) Component of rock weight **(b) Force component of rock**

Figure 3 – Forces on rock located within the channel bank

The weight component down the face of the bank slope is given by Equation 116:

$$W.\sin(\alpha) \tag{116}$$

The resultant force (F) acting on the rock is given by Equation 117:

$$F^2 = W^2.\sin^2(\alpha) + (\text{drag force})^2 \tag{117}$$

The drag force is represented by Equation 118:

$$F_d = a.\tau_o \tag{118}$$

The resisting force (F_R) as suggested by Victoria Rivers (1991), Chow (1959) and Henderson (1966) is given by Equation 119:

$$F_R = W.\cos(\alpha)\ .\ \tan(\Phi) \tag{119}$$

where:

- α = bank slope relative to the channel bed [degrees]
- F = resultant force on rock [N]
- F_d = drag force of rock [N]
- a = effective surface area [m^2]
- F_R = resisting force [N]
- Φ = angle of repose of rock [degrees]

If the design safety factor (SF_F) is adopted as the ratio of resisting force (F_R) divided by the resultant active force (F), then:

$$SF_F = F_R / F \tag{120}$$

$$SF_F = \frac{W.\cos(\alpha).\tan(\phi)}{\sqrt{(W^2.\sin^2(\alpha) + a^2.\tau_o{}^2)}} \tag{121}$$

For a flat bed (i.e. $\alpha = 0^o$) Shield's equation gives:

$$\tau_c = F^*.\ \rho_w.g.d_{50}\,(s_r - 1) \tag{122}$$

If a safety factor of 1.0 is chosen (i.e. critical conditions), then:

$$SF_F = \frac{W.\cos(0^o).\tan(\phi)}{\sqrt{(W^2.\sin^2(0^o) + a^2.\tau_c{}^2)}} = 1.0 \tag{123}$$

therefore;

$$a.\tau_c = W.\tan(\phi) \tag{124}$$

Equation 124 assumes that the effective shear stress surface area (a) can be directly related to the angle of repose, which is not necessarily the case.

Combining equations 122 and 124 gives:

$$a = \frac{W.\tan(\phi)}{F^{*}.\rho_w.g.d_{50}.(s_r - 1)} \quad (125)$$

Also, Equations 21 and 22 provide the maximum shear stress on the bed and banks.

Maximum shear stress on bed: $$\tau_o = \rho_w.g.y.\sin(\varphi) \quad (126)$$

Maximum shear stress on bank: $$\tau_o = (0.75)\rho_w.g.y.\sin(\varphi) \quad (127)$$

Technical Note 2:

In the original Victoria Rivers (1991) derivation it was assumed that sin(φ) in Equations 126 and 127 could be replaced by S_e however, this is only appropriate for low channel bed and bank slopes. It is also noted that Victoria Rivers indicated that shear stress on the bed was defined by, $\tau_o = (0.97)\ \rho_w.g.y.S$ (different from above).

The following presents equations different from those presented within the original Victoria Rivers (1991) publication. Specifically, the term S_e has been replaced by sin(φ), and the maximum shear stress expression for the channel bed, presented above as Equation 126 is adopted instead of the expression noted within the above technical note.

Equation 121 now becomes:

$$SF_F = \frac{W.\cos(\alpha).\tan(\phi)}{\sqrt{(W^2.\sin^2(\alpha) + \left(\frac{\tau_o.W.\tan(\phi)}{F^{*}.\rho_w.g.d_{50}(s_r - 1)}\right)^2}} \quad (128)$$

which simplifies to:

$$SF_F = \frac{\cos(\alpha).\tan(\phi)}{\sqrt{(\sin^2(\alpha) + \left(\frac{\tau_o.\tan(\phi)}{F^{*}\rho_w.g.d_{50}(s_r - 1)}\right)^2}} \quad (129)$$

(a) Rock on channel bed

Given that the maximum shear stress on the channel bed is (Equation 126):

$$\tau_o = \rho_w.g.y.\sin(\varphi) \quad (130)$$

Equation 97 becomes:

$$SF_F = \frac{\cos(\alpha).\tan(\phi)}{\sqrt{(\sin^2(\alpha) + \left(\frac{\rho_w.g.y.\sin(\varphi).\tan(\phi)}{F^{*}.\rho_w.g.d_{50}(s_r - 1)}\right)^2}} \quad (131)$$

$$SF_F = \frac{\cos(\alpha).\tan(\phi)}{\sqrt{(\sin^2(\alpha) + \left(\frac{y.\sin(\varphi).\tan(\phi)}{F^{*}.d_{50}(s_r - 1)}\right)^2}} \quad (132)$$

Also, for rock on the channel bed, α = 0°, thus:

$$SF_F = \frac{\cos(0^o).\tan(\phi)}{\sqrt{(\sin^2(0^o) + \left(\frac{y.\sin(\varphi).\tan(\phi)}{F^{*}.d_{50}(s_r - 1)}\right)^2}} \quad (133)$$

$$SF_F = \frac{\tan(\phi)}{\sqrt{\left(\frac{y.\sin(\varphi).\tan(\phi)}{F^*.d_{50}.(s_r - 1)}\right)^2}} \tag{134}$$

$$SF_F = \frac{F^*.d_{50}(s_r - 1).\tan(\phi)}{y.\sin(\varphi).\tan(\phi)} \tag{135}$$

$$SF_F = \frac{F^*.d_{50}(s_r - 1)}{y.\sin(\varphi)} \tag{136}$$

Rock on bed of channel/chute:

$$d_{50} = \frac{SF_F.y.\sin(\varphi)}{F^*(s_r - 1)} \tag{137}$$

For a low-gradient channel, $\sin(\varphi) \approx S_e$ and Equation 137 is effectively the same as the formula presented within Victoria Rivers (1991) publication.

It should be noted that Equation 137 is independent of the angle of repose of the rock (Φ).

(b) Rock on channel bank

Given that the maximum shear stress on the channel bank is (Equation 95):

$$\tau_o = (0.75).\rho_w.g.y.\sin(\varphi) \tag{138}$$

Thus for rock on the **banks** of a channel Equation 97 becomes:

$$SF_F = \frac{\cos(\alpha).\tan(\phi)}{\sqrt{(\sin^2(\alpha) + \left(\frac{(0.75)\rho_w.g.y.\sin(\varphi).\tan(\phi)}{F^*.\rho_w.g.d_{50}(s_r - 1)}\right)^2}} \tag{139}$$

$$SF_F = \frac{\cos(\alpha).\tan(\phi)}{\sqrt{(\sin^2(\alpha) + \left(\frac{(0.75).y.\sin(\varphi).\tan(\phi)}{F^*.d_{50}(s_r - 1)}\right)^2}} \tag{140}$$

This can be rearranged to produce:

$$d_{50} = \frac{(0.75).y.\sin(\varphi)}{F^*.\cos(\alpha).(s_r - 1)\sqrt{\left(\frac{1}{SF_F^{\,2}} - \frac{\tan^2(\alpha)}{\tan^2(\phi)}\right)}} \tag{141}$$

However, Equation 141 is based on the assumption that the resisting force (F_R) is given by Equation 119, which is not considered appropriate for very steep chutes. On steep chutes the effective maximum slope of the banks of the chute is a function of both the slope of the bank relative to the chute bed (α) plus the slope of the chute bed relative to the horizontal (θ).

It is noted that when the slope of the channel is significant, the gravitational force acting on a rock located on the channel bank is subject to a slope greater than the bank slope (i.e. the channel is now a complex three dimensional structure). The effective slope (β) of the rock weight component on a double slope formed by a bank slope (α) and a channel bed slope (θ) is given by Equation 142.

$$\tan^2(\beta) = \tan^2(\alpha) + \tan^2(\theta) \tag{142}$$

Therefore, the resisting force (F_R) is now given by Equation 143:

$$F_R = W.\cos(\beta).\tan(\Phi) \tag{143}$$

Thus Equation 141 now becomes:

Rock on bank of chute: $$d_{50} = \frac{(0.75).y.\sin(\varphi)}{F^*.(s_r - 1)\sqrt{\left(\frac{\cos^2(\beta)}{SF_F{}^2} - \frac{\sin^2(\alpha)}{\tan^2(\phi)}\right)}} \qquad (144)$$

If a low-gradient channel is assumed (i.e. $\sin(\varphi) \approx S_e$, and $\cos(\beta) \approx \cos(\alpha)$), then Equation 112 becomes:

(rock on channel bank) $$d_{50} = \frac{(0.75).y.S_e}{F^*.\cos(\alpha).(s_r - 1)\sqrt{\left(\frac{1}{SF_F{}^2} - \frac{\tan^2(\alpha)}{\tan^2(\phi)}\right)}} \qquad (145)$$

For a low gradient channel, Equation 145 is effectively the same as the formula originally presented by Victoria Rivers (1991).

4.5 Rock sizing for rock lined embankment experiencing overtopping flows

4.5.1 Isbash (1936)

As previously reported in Section 4.3.1, Isbash investigated the stability of rounded rock dumped into flowing rivers to for dams. This work focused on sizing individual stones (located on the downstream face of a dam) to resist displacement due to overtopping flow and percolation through the embankment.

The results of Isbash (1936) are presented in McLaughlin Water Engineers (1986) and reproduced below as Equation 146.

$$d_{50} = \frac{V^2}{2.g.K^2.(s_r - 1)} \qquad (146)$$

where:

K = equation constant based on flow conditions

Isbash recommended K = 0.86 for high turbulent river flow and K = 1.2 for low turbulent river.

4.5.2 Stephenson, D. (1979)

Stephenson (1979) performed a stability analysis for rocks placed on the downstream face of a rock embankment subject to overtopping flows. The results of Stephenson are reported in Abt and Johnson (1991).

$$d_{50} = \left[\frac{q(\tan\theta)^{7/6}\, n_p{}^{1/6}}{C\, g^{1/2}\, [(1 - n_p)(s_r - 1)(\cos\theta)(\tan\phi - \tan\theta)]^{5/3}}\right]^{2/3} \qquad (147)$$

where:

- q = threshold flow per unit width [$m^3/s/m$]
- n_p = porosity of rock embankment
- C = coefficient (0.22 for gravel and pebbles, 0.27 for crushed stone)
- s_r = relative density of rock
- θ = slope of rock (embankment) face [degrees]
- Φ = angle of repose of rock (angle of friction) [degrees]

Complete failure of the embankment is reported to occur when the unit discharge increased by 8% for crushed stone.

4.5.3 Abt, S.R. and Johnson, T.L. (1991)

Abt, S.R. and Johnson, T.L. (1991) investigated critical rock size (i.e. failure condition) for embankments subjected to overtopping flows. Bank slopes (S) of 1 to 20% were investigated.

This paper also indicates that Stephenson (1979) showed that due to the added interlocking of crushed rock, the required rock size (prior to threshold flow or incipient stone movement) is 87% of that required for natural rounded rock (river stone).

Abt and Johnson reported that “angular stone tends to interlock or wedge and subsequently resist sliding and rolling. In addition, fewer fines are required to fill the voids of crushed material compared with a similarly graded rounded stone.”

The literature research showed that riprap designed to resist overtopping flows is a function of:

- relative rock size;
- irregularity of the rock (round of fractured quarry stone);
- the hydraulic gradient;
- the discharge (or unit discharge).

Abt and Johnson concluded that the critical rock size (for **angular crushed rock**), at which point failure begins to occur, is given by the following equation:

(condition of total failure) $$d_{50,failure} = 0.5026\ S_o^{\ 0.43}\ q^{\ 0.56} \qquad (148)$$

where:

$d_{50,critical}$ = mean rock size (specific gravity = 2.65) at point of failure [m]

S_o = slope of rock protected embankment (range 1 to 20%) [m/m]

q = flow per unit width down the embankment [m^3/s/m]

It is important to note that the bed slope (S_o) is used in this equation, not the energy slope.

For incipient motion conditions the above equation becomes:

(incipient rock movement) $$d_{50,critical} = 0.5950\ S_o^{\ 0.43}\ q^{\ 0.56} \qquad (149)$$

For design purposes a safety factor of 1.2 is recommended, thus the above equation becomes:

(design purposes) $$d_{50} = 0.595\ (SF)\ S_o^{\ 0.43}\ q^{\ 0.56} \qquad (150)$$

(for SF = 1.2) $$d_{50} = 0.714\ S_o^{\ 0.43}\ q^{\ 0.56} \qquad (151)$$

The above rock-sizing equations are based on a coefficient of uniformity, $C_u = d_{60}/d_{10} = 2.15$.

Abt and Johnson reported that for rounded rock, failure occurred at flow rates 32 to 45% lower than that for angular rock at a 10% embankment slope. This would result in a 17 to 23% increase in required rock size, i.e. angular rock would need to be 81 to 85% of rounded rock size, which is similar to the results of Stephenson (1979).

4.6 Rock sizing for rock-lined chutes

4.6.1 McLaughlin Water Engineers (Denver Guidelines, 1986)

The recommended design equation for rock on the bed of steep chutes as presented by McLaughlin Water Engineers is reproduced below as Equation 152.

$$d_{50} = \frac{\tau}{F * \rho_w . g . (s_r - 1) . \cos(\theta) . \left(\frac{1}{SF_F} - \frac{\tan(\theta)}{\tan(\phi)} \right)} \qquad (152)$$

Equation 152 was further developed by McLaughlin Water Engineers, but it is the author’s opinion that their derivation contains errors as discussed below.

Technical Note 3:
McLaughlin Water Engineers assumed that shear stress (τ) is represented by the following equation:

$$\tau = \rho_w . g . y . S_e$$

But this assumes a low-gradient chute slope where S_e = tan (φ) ≈ sin(φ) which is **not** the case for a steep chute.

If the correct (in the author's opinion) expression for bed shear stress is inserted into Equation 152, then a rock-sizing equation in the form of Equation 153 is developed.

Bed rock (non-uniform flow):
$$d_{50} = \frac{y.\sin(\varphi)}{F^*.\cos(\theta).(s_r - 1).\left(\frac{1}{SF_F} - \frac{\tan(\theta)}{\tan(\phi)}\right)} \quad (153)$$

Noting that for non-uniform flow:
$$\sin(\varphi) \neq \sin(\theta) \quad (154)$$

McLaughlin Water Engineers also recommend the following parameters:

F^* = 0.091

SF_F = 1.5

Φ = 42 degrees (assumed for large angular rock)

If uniform flow conditions are assumed down the chute, such that the energy slope is parallel with the bed slope (i.e. $S = S_o = S_e$, and sin(φ) = sin(θ)), then Equation 153 becomes:

Bed rock (uniform flow):
$$d_{50} = \frac{y.\tan(\theta)}{F^*.(s_r - 1).\left(\frac{1}{SF_F} - \frac{\tan(\theta)}{\tan(\phi)}\right)} \quad (155)$$

McLaughlin Water Engineers recommended that rock size should be based on actual flow depth (y) above the rock rather than average flow depth (y_{ave}), or the hydraulic radius (R). Effectively the design is based on a 2-dimensional (2D) flow profile even though actual flow conditions may be highly 3-dimensional. The flow per unit width (q) is based on the hydraulic analysis of actual flow conditions at the crest (i.e. adopting the actual 3D crest profile).

4.7 Effects of the distribution of rock size on rock stability

The distribution of rock size assumed throughout this report for the conversion of reported rock-sizing equation based on d_X rock size to an equivalent d_{50} rock size is presented in Table 15.

Table 15 – Assumed distribution of rock size used in steep chutes

Rock size ratio	Assumed distribution value
d_{100}/d_{50}	2.00
d_{90}/d_{50}	1.82
d_{75}/d_{50}	1.50
d_{65}/d_{50}	1.28
d_{40}/d_{50}	0.75
d_{33}/d_{50}	0.60
d_{10}/d_{50}	0.55

The distribution of rock size can also be described by the coefficient of uniformity, $C_u = d_{60}/d_{10}$. Poorly graded rock has a low value of C_u (say, C_u<2), while well-graded rock has a higher value of C_u (say, C_u>2.5). Witter & Abt (1990) report that the recommended values of C_u specified by various published design guidelines generally falls in the range 1.1 to 2.70, but a typical value is around 2.1.

Witter & Abt investigated the effects of variations in the distribution of rock sizes on the stability of riprap. Their conclusions are summarised below:

- Poorly graded riprap has a high overall stability than well-graded riprap.
- Following the displacement of individual surface rocks, well-graded riprap tends to fill in the voids left by the displaced rocks through the movement of small particles washed from above, a process referred to as 'healing'.
- Following the displacement of individual surface rocks, poorly graded riprap exhibits very little subsequent movement (rearranging) of rocks up until the point of mass failure.
- Failure of well graded riprap occurs more slowly than poorly graded riprap due to the 'healing' action performed by the smaller rocks.
- Failure of poorly graded riprap begins with many surface rocks moving very suddenly.
- The aftermath of a keystone failure is a deep longitudinal scour in the embankment.
- A conservative approach to the sizing of rock should apply if the coefficient of uniformity (C_u) exceeds 3.0 [observation of author following review of published test results].

In addition to the above, Witter & Abt report that Ahmed (1989) indicated that for rock placement on channel banks, poorly graded rock ($Cu \approx 1.1$) had a critical discharge 8% greater than well - graded rock ($Cu \approx 2.2$). This would equate to allowing a 5% reduction in rock size for poorly graded rock compared to well graded rock when the rock is placed on the side slopes of a channel or chute.

5. Rock chute case studies and laboratory testing

5.1 Drop structures within the Denver, Colorado region

McLaughlin Water Engineers (1986) reported on field observations of various failed or stressed drop structures within the Urban Drainage and Flood Control District 69, Denver, Colorado, USA. A summary of the filed observations is presented in Table 16.

Table 16 – Case studies of sloping loose rock drop structures reported by McLaughlin Water Engineers (1986)

Site	Unit flow (m^3/s/m)	Slope (H:1)	Drop height (m)	Mean rock size (mm)	Comments
Bear Canyon Creek Drop 5	1.04	4:1		450	Movement of few rocks
Cherry Creek	1.49	7:1		300 to 350	Expected failure point
Liberty Hill	1.96	4.5:1		< 600	Significant rock movement
Bear Canyon Creek Drop 9	2.08	4:1		380	Massive failure
Little Dry Creek, Westminster	2.34	5.35:1		450+	Some movement of small rocks
Lone Tree	2.74	11.5:1	0.6 to 0.9	300 to 380	Slight rock movement
Little Dry Creek, Georgetown	3.02	4.5:1		< 600	Some movement
Massey Draw	3.01	4:1	1.8	450	Rock clearly undersized for flow
Englewood Dam	4.38	8:1		450	Significant rock movement

5.2 Gay Street drop structure, Coopers Plains, Brisbane

Photos 3 & 4 show a rock-lined grade control structure (drop structure) constructed within a modified watercourse in the southern suburbs of Brisbane. This structure experienced significant damage

Design conditions:

- Design flow rate = 105 m^3/s
- Design unit flow rate = 4.5 m^3/s/m
- Chute slope = 15:1
- Assumed distribution of rock size, d_{50}/d_{90} = 0.5
- Design required mean rock size, d_{50} = 750 mm with d_{10} = 600 mm
- Trapezoidal approach channel and chute, with 1m wide, grouted rock low-flow channel passing through the elevated weir crest
- The low-flow channel does not continue down the face of the chute
- Crest width (at invert) = 23 m
- Design bank slope of 2.5:1 one side and 3.5:1 other side (at crest)
- Drop height from crest to downstream channel invert = 2.54 m

Post storm survey of damaged drop structure:

- Constructed weir crest profile was surveyed (Figure 4)
- Mean rock size estimated at 600 mm +/- 100 mm
- Movement in the 600 mm rock appears to have been a direct response to the loss of the smaller rocks
- Rocks moved well down the channel by flow were up to 400 mm, but larger rock were displaced as a result of total failure of the structure
- Estimate the critical rock size to be 500 mm (i.e. average of 400 mm and 600 mm)

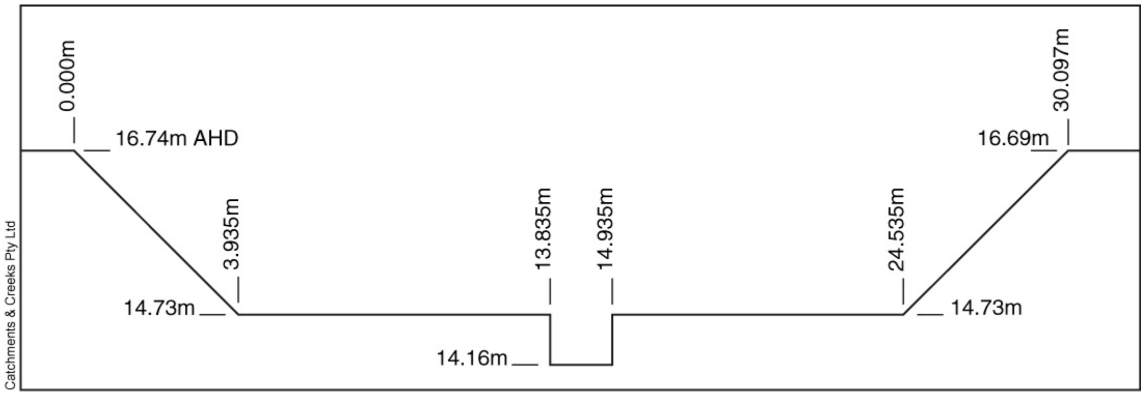


Figure 4 – Cross section (xs573) of chute crest, Gay Street drop structure

Destructive flow event, 9 March 2001:

Estimated peak discharge = 105 m^3/s (approx 1 in 30 year event), but could be in the range 92 to 120 m^3/s

Constructed weir crest profile was surveyed (Figure 4) and used in the hydraulic analysis

Critical depth at crest (xs573) at 105 m^3/s = 15.99 m AHD, flow area = 31.33 m^2, top width = 27.76 m, average velocity = 3.35 m/s

Table 17 – HecRas analysis of 9 March 2001 storm

Cross-section	Chainage	Water elevation (AHD)
XS 653	653 m	16.77 m
XS 573 (crest)	573 m	15.99 m
XS 453	453 m	14.7 m

Storm conditions at critical cross sectional flow conditions (XS563):

Energy slope = 0.06676

Flow depth = 1.293 m

Calculated unit flow rate = 5.0 $m^3/s/m$

Estimated Manning's roughness = 0.081

Calculated velocity = 3.45 m/s

Table 18 – Case study of Gay Street, Coopers Plains, Brisbane

Unit flow ($m^3/s/m$)	Slope (H:1)	Drop height (m)	Mean rock size (mm)	Comments
5.0	15:1	2.5	600–400	Assumed critical rock size = 500 mm

Photo 3 – Failed drop structure seven month after 1 in 30 yr storm

Photo 4 – Failed structure with concrete weir visible and undamaged

5.3 Physical model studies reported by Abt and Johnson (1991)

Abt and Johnson (1991) reported on flume studies of rock embankments experiencing overtopping flows.

Table 19 – Summary of laboratory test results by Abt and Johnson (1991)

Rock shape	d_{50}	Slope	flow rate (failure)	Specific weight	d_{50}/d_{90}	Manning	Depth
	(m)	(–)	(m3/s/m)	–	–	–	(m)
	Reported values				Calculated values		
Angular	0.104	0.20	**0.168**	2.65	0.249	0.169	0.193
Angular	0.130	0.20	**0.331**	2.65	0.288	0.139	0.257
Angular	0.157	0.20	**0.412**	2.65	0.289	0.146	0.303
Angular	0.056	0.20	**0.046**	2.72	0.258	0.162	0.086
Rounded	0.102	0.20	**0.088**	2.65	0.198	0.254	0.166
Rounded	0.102	0.20	**0.088**	2.65	0.198	0.254	0.166
Angular	0.051	0.10	**0.079**	2.72	0.215	0.145	0.138
Angular	0.051	0.10	**0.093**	2.72	0.215	0.139	0.147
Angular	0.051	0.10	**0.103**	2.72	0.215	0.135	0.155
Rounded	0.102	0.10	**0.185**	2.50	0.240	0.154	0.238
Rounded	0.102	0.10	**0.194**	2.50	0.240	0.153	0.243
Angular	0.102	0.10	**0.326**	2.65	0.198	0.163	0.347
Angular	0.102	0.10	**0.352**	2.65	0.198	0.160	0.359
Rounded	0.051	0.10	**0.064**	2.72	0.094	0.367	0.210
Angular	0.102	0.10	**0.383**	2.65	0.198	0.156	0.373
Angular	0.056	0.20	**0.421**	2.72	0.258	0.093	0.236
Angular	0.026	0.20	**0.103**	2.72	0.301	0.073	0.087
Angular	0.026	0.01	**0.139**	2.72	0.301	0.049	0.202
Angular	0.026	0.10	**0.033**	2.72	0.301	0.089	0.061
Angular	0.026	0.10	**0.032**	2.72	0.301	0.090	0.060
Angular	0.026	0.10	**0.029**	2.72	0.301	0.092	0.057
Angular	0.026	0.10	**0.039**	2.72	0.301	0.085	0.066
Angular	0.056	0.10	**0.104**	2.72	0.258	0.120	0.146
Angular	0.056	0.10	**0.116**	2.72	0.258	0.118	0.154
Angular	0.056	0.10	**0.116**	2.72	0.258	0.118	0.154
Angular	0.056	0.08	**0.168**	2.72	0.258	0.104	0.191

Table 20 – Average and range of test results and parameters for the laboratory testing carried out by Abt and Johnson (1991)

	Specific gravity	Mean rock size	Slope	Unit flow at failure	Assumed depth	y/d_{50}	Critical Shields [1]
	–	(mm)	(m/m)	(m^3/s/m)	(m)	–	–
Minimum	2.50	26	0.01	0.029	0.057	1.54	0.040
Average	2.68	69	0.127	0.162	0.184	2.88	0.194
Maximum	2.72	157	0.2	0.421	0.373	7.77 [2]	0.311

[1] The equivalent Shield's parameter has been determined using the equation presented by McLaughlin Water Engineers based on a safety factor, SF = 1.0 (i.e. critical conditions).

[2] The second highest value was, y/d_{50} = 4.21.

Tables 19 & 20 represent a further analysis of the data presented by Abt and Johnson. This analysis was based on the following assumptions and calculations:

- Distribution of rock size (d_{50}/d_{90}) was estimated from the reported (d_{84}/d_{16}) value.
- Manning's roughness was determined through the application of Equation 36.
- Flow depth was calculated to achieve the reported unit flow rate for the assumed Manning's roughness, effective hydraulic radius assumed equal to the flow depth, and energy slope equal to the embankment slope.

5.4 Physical model studies reported by Peirson et al. (2008)

Peirson et al. (2008) conducted physical model studies on both randomly dumped and placed rock layered directly on the bed of a tilting flume subject to overtopping flows. The rock and placement properties are summaries in Table 21.

Table 21 – Summary of rock and placement properties

Description	Sandstone		Sandstone		Basalt	
Specific gravity (s_r)	2.29		2.37		2.64	
Mean diameter (d_{50}) [mm]	76		109		94	
Coefficient of uniformity (C_u)	1.18		1.14		1.21	
Rock placement properties:						
Configuration	Random	Placed	Random	Placed	Random	Placed
Packing number [rocks/m^2/layer]	145	187	71	90	93	117
Porosity	0.47	0.39	0.44	0.38	0.47	0.37
Depth of two layers of rock [mm]	121	141	174	204	151	184
Unit density of two layers [kg/m^3]	1230	1410	1320	1450	1405	1717
Angle of repose [1] [degrees]	44	49	50	49	48	53

[1] Determined from an average of three observations of the angle of tilt required to induce rock displacement on a test ramp with an underlayer of 40 mm basalt fixed to the ramp floor.

Failure was judged as the condition where the rock armour exposed the filter material. The total discharge being the sum of the flow carried within the rock blanket (thus subject to the thickness and porosity of the rock layer) and the flow over the rock blanket. A summary of the test results is presented in Table 22.

Table 22 – Summary of test results

	Sandstone (76 mm)			Sandstone (109 mm)			Basalt (94 mm)		
Bed slope (m/m)	**0.2**	**0.3**	**0.4**	**0.2**	**0.3**	**0.4**	**0.2**	**0.3**	**0.4**
Random placement									
$q_{failure}$ [m^2/s]	0.118	0.084	0.054	0.233	0.156	0.156	0.194	0.183	0.161
Placed rock									
$q_{failure}$ [m^2/s]	0.156	0.147	0.124	0.308	0.245	0.201	N/A	0.271	0.256
$q_{(placed)}/q_{(random)}$	1.32	1.75	2.29	1.32	1.57	1.29	N/A	1.48	1.59

Other outcomes from this laboratory testing include:

- A typical 30% increase in the design flow for placed rock compared to randomly placed rock, but an approximate 35% increase in the density of rock used to form two layers of placed rock.
- Average thickness of two layers of rock were 1.60(d_{50}) for randomly placed rock and 1.86(d_{50}) for placed rock.
- Flow must not exceed 50% of the failure discharge if substantial rearrangement of the armour rock is to be avoided.

6. Determination of a new equation for sizing rock based on the combined field and laboratory data

Various rock-sizing equations obtained from the literature search where tested on the field and laboratory test data presented in the previous section to determine a best-fit equation.

The literature search revealed that the equations presented for sizing rock in deepwater, low gradient channels generally take the form of the following three equations:

$$d_{50} = \frac{SF.R.S_e}{F*(s_r - 1)} \quad (156)$$

$$d_{50} = \frac{SF.V^2}{2.g.K^2.(s_r - 1)} \quad (157)$$

$$d_{50} = \frac{0.031(SF)}{(s_r - 1)^{1.25}}\left(\frac{V^{2.5}}{y^{0.25}}\right) \quad (158)$$

In order for the above deep-water, low-gradient equations to blend seamlessly with the shallow-water, steep-gradient equation presented by Abt and Johnson, it is desirable for the above equations to be presented in a form similar to Equation 159:

$$d_{50,critical} = \frac{K.K_1.K_2.S_e{}^A.q^B}{(s_r - 1)} \quad (159)$$

where:

$d_{50,cri}$	=	critical (SF = 1) mean rock size [m]
K	=	equation constant
K_1	=	correction factor for rock shape
	=	1.0 for angular (fractured) rock, 1.36 for rounded rock (i.e. smooth, spherical rock)
K_2	=	correction factor for rock grading
	=	0.95 for poorly graded rock ($C_u = d_{60}/d_{10} < 1.5$), 1.05 for well graded rock ($C_u > 2.5$), otherwise $K_2 = 1.0$ ($1.5 < C_u < 2.5$)
S_e	=	slope of energy line at given cross-section [m/m]
q	=	unit flow rate based on local flow depth at a given location [m^3/s/m]
s_r	=	specific gravity of rock

If we consider 2D flow conditions where the hydraulic radius (R) is equal to flow depth (y), then the following applies:

$$V = \frac{R^{2/3}\,S_e{}^{1/2}}{n} \quad (160)$$

and

$$q = \frac{R^{5/3}\,S_e{}^{1/2}}{n} \quad (161)$$

Therefore Equations 143 to 145 can be transformed as follows:

$$d_{50} = \frac{constant.R.S_e}{(s_r - 1)} \quad (162)$$

$$d_{50} = \frac{constant.R^{4/3}.S_e}{(s_r - 1)} \quad (163)$$

$$d_{50} = \frac{constant.R^{17/12}.S_e{}^{5/4}}{(s_r - 1)^{1.25}} \quad (164)$$

Thus Equations 162 to 164 can be presented in the form:

$$d_{50} = \frac{\text{constant}.R^{C}.S_{e}^{D}}{(s_r - 1)^{E}} \qquad (165)$$

where:

$$1.0 < C < 1.42$$

$$1.0 < D < 1.25$$

$$1.0 < E < 1.25$$

and:

$$C = \frac{5B}{3} \qquad (166)$$

$$D = A + 0.5B \qquad (167)$$

Therefore, to obtain an equation that works for shallow water, deep water, and low and steep gradient bed conditions, it would 'appear' desirable for the power constants A and B (Eqn 159) to fall within the range:

$$0.575 < A < 0.95 \qquad (168)$$

$$0.6 < B < 0.85 \qquad (169)$$

A best-fit analysis of the field and laboratory data produced the following equation:

(Uniform flow)

$$d_{50,critical} = \frac{0.787.K_1.K_2.S_o^{0.35}.q^{0.63}}{(s_r - 1)} \qquad (170)$$

The data set, however, contains some field observations from Denver where the observed rock size was noted as being clearly undersized, or the discharge was significantly greater than the design storm. This puts into question the suitability of using this data in the development of a new best-fit equation. The three sites in question are listed in Table 23.

Table 23 – Questionable field data

Site	Comments recorded in original report
Bear Canyon Creek Drop 5	• Movement of few rocks • Rocks clearly smaller than that specified.
Bear Canyon Creek Drop 9	• Massive failure
Massey Draw	• Rock clearly undersized for flow • Rocks clearly smaller than that specified. • Rocks are poorly graded with subgrade open to flow

The above three data points were removed from the data set and the best-fit analysis was repeated. Based on this reduced data set the following revised best-fit equation was developed.

(Uniform flow)

$$d_{50,critical} = \frac{0.81.K_1.K_2.S_o^{0.36}.q^{0.63}}{(s_r - 1)} \qquad (171)$$

Plots of the observed d_{50} rock size compared to that calculated by the relevant best-fit equations are presented as Figure 5 and 6.

As can be seen from Equation 170, the power constant applied to the unit flow rate (q) is 0.63, which is inside the expected range of 0.6 to 0.85 (Eqn 169); however, the power constant applied to the bed slope (S_o) of 0.35 is below the expected range of 0.575 to 0.95 (Eqn 168).

Using Equation 171 developed from the reduced data set, the power constant applied to the bed slope (S_o) of 0.36 is also below the expected value of 0.575 to 0.95; however, the power constant applied to the unit flow rate (q) remains at 0.63, which is inside the expected range of 0.6 to 0.85.

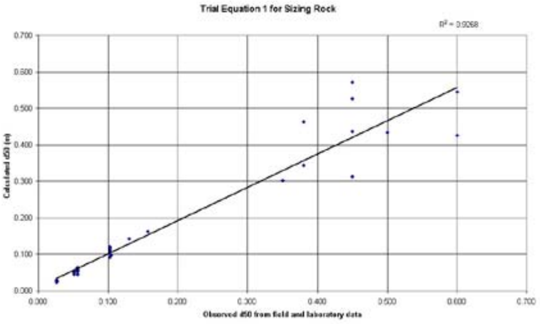


Figure 5 – Observed rock size vs calculated rock size using full data set

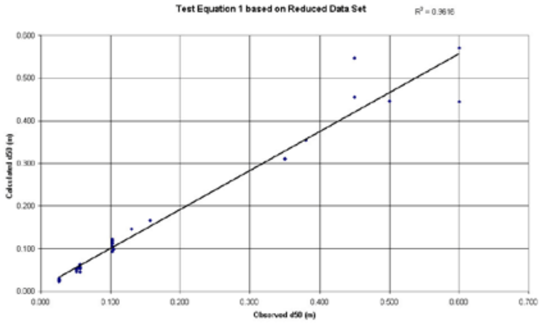


Figure 6 – Observed rock size vs calculated rock size using reduced data set

The above results imply that the bed slope (S_o) may not be as influential in determining rock size as previously thought. It is noted that both energy slope and bed slope are considered here because the equations analysed assume uniform flow conditions where energy slope and bed slope are equal. However, logic would suggest that it is in fact the reduced influence of bed slope. Possible reasons for this reduced influence of bed slope include:

- The effects of interlocking between individual rocks resulting from the weight of up-slope rocks impinge upon the lower rocks thus increasing their stability.
- The effects of air entrainment on steep slopes.
- Those equations based on shear stress assume that under uniform flow conditions (i.e. the flow reached a constant velocity where energy slope equals bed slope) the riprap exerts a force on the water equal to the weight of the water. However, this assumption becomes increasingly questionable with increasing bed slope.

Equations 170 and 171, however, do not necessarily take the preferred equation format because neither equation incorporates the angle of repose of the rock, which is considered significant at steep bed slopes. Instead, the effects of the angle of repose are directly incorporated into the bed slope power factor through laboratory testing of steep rock-lined chutes and spillways. Also, neither equation is appropriate for non-uniform flow when bed slope (S_o) and energy slope (S_e) are not equal.

It was further noted that the majority of the field and laboratory data is based on shallow water conditions relative to the size of the rocks. If the equations developed from this project are going to be flexible enough to be used in both deep and shallow water conditions, then it was considered necessary to supplement the existing data set with some information (data points) appropriate for deepwater conditions.

To determine these additional data points, the work of the following authors was considered:

- McLaughlin Water Engineers (Denver Guidelines, 1986)
- Maynord (1978)
- Maynord, Ruff & Abt (1989)
- Abt, S.R. and Johnson, T.L. (1991)

After comparing the rock sizes determined from the various equations presented by the above authors it was decided that a set of a low bed slope, deepwater data points could be obtained from an average of the standard Shield's equation with a Shield's parameter of 0.07, and the recommended design equation presented by Maynord (1978).

Table 24 provides a summary of key parameters associated with the original field and laboratory data points. Tables 25 and 26 provide a summary of key parameters associated with the additional low-gradient, deepwater data points.

Table 24 – Adopted data parameters used in testing the equations

Location	d_{90} (m)	Manning's roughness	Local velocity (m/s)	Unit discharge	K1	K2
Bear No.5 [1]	0.910	0.121	2.38	1.04	1.00	1.00
Cherry Ck	0.796	0.098	2.55	1.49	1.00	1.00
Liberty Hill	1.250	0.121	2.92	1.96	1.00	1.00
Bear No.9 [1]	1.160	0.125	2.73	2.08	1.00	1.00
Little Dry Ck	1.150	0.112	2.94	2.34	1.00	1.00
Lone Tree	0.930	0.087	2.95	2.74	1.00	1.00
Georgetown	1.520	0.125	3.22	3.02	1.00	1.00
Massey D. [1]	1.450	0.130	3.04	3.01	1.00	1.00
Englewood	1.340	0.102	3.42	4.38	1.00	1.00
Gay St	1.300	0.086	3.56	5.00	1.00	1.00
Sandstone	0.095	0.050	1.59	0.12	1.00	0.95
Sandstone	0.095	0.056	1.45	0.08	1.00	0.95
Sandstone	0.095	0.065	1.22	0.05	1.00	0.95
Sandstone	0.136	0.051	2.05	0.23	1.00	0.95
Sandstone	0.136	0.059	1.81	0.16	1.00	0.95
Sandstone	0.136	0.061	1.94	0.16	1.00	0.95
Basalt	0.118	0.050	1.94	0.19	1.00	0.95
Basalt	0.118	0.053	2.07	0.18	1.00	0.95
Basalt	0.118	0.056	2.06	0.16	1.00	0.95
Angular	0.462	0.117	1.28	0.17	1.00	1.00
Angular	0.580	0.117	1.54	0.33	1.00	1.00
Angular	0.675	0.119	1.69	0.41	1.00	1.00
Angular	0.205	0.102	0.86	0.05	1.00	1.00
Rounded	0.444	0.124	1.09	0.09	1.36	1.00
Rounded	0.434	0.122	1.11	0.09	1.36	1.00
Angular	0.242	0.088	0.96	0.08	1.00	1.00
Angular	0.256	0.088	1.00	0.09	1.00	1.00
Angular	0.274	0.090	1.01	0.10	1.00	1.00
Rounded	0.396	0.092	1.31	0.19	1.36	1.00
Rounded	0.408	0.093	1.32	0.19	1.36	1.00
Angular	0.631	0.105	1.49	0.33	1.00	1.00
Angular	0.646	0.105	1.52	0.35	1.00	1.00
Rounded	0.521	0.122	0.92	0.06	1.36	1.00
Angular	0.672	0.105	1.55	0.38	1.00	1.05
Angular	0.182	0.055	0.99	0.42	1.00	1.00
Angular	0.063	0.044	0.62	0.10	1.00	1.00
Angular	0.106	0.042	0.82	0.14	1.00	1.00
Angular	0.093	0.069	0.71	0.03	1.00	1.00
Angular	0.093	0.070	0.69	0.03	1.00	1.00
Angular	0.086	0.068	0.69	0.03	1.00	1.00
Angular	0.100	0.070	0.74	0.04	1.00	1.00
Angular	0.236	0.084	1.05	0.10	1.00	1.00
Angular	0.252	0.085	1.07	0.12	1.00	1.00
Angular	0.252	0.085	1.07	0.12	1.00	1.00
Angular	0.279	0.081	1.16	0.17	1.00	1.00

[1] Data point not considered to be representative of the critical rock size and ultimately removed from the final best-fit equation analysis.

Table 25 – Adopted data parameters for the additional best-guess data points

d_{50}	Slope	flow rate (failure)	Specific weight	d_{50}/d_{90}	Manning	Depth
(m)	(–)	(m3/s/m)	–	–	–	(m)
0.074	0.01	2.52	2.6	0.8	0.031	1.00
0.194	0.02	3.04	2.6	0.8	0.047	1.00
0.149	0.01	7.11	2.6	0.8	0.035	2.00
0.387	0.02	8.60	2.6	0.8	0.052	2.00
0.396	0.05	4.67	2.6	0.8	0.065	1.20
0.223	0.01	13.07	2.6	0.8	0.037	3.00
0.581	0.02	15.80	2.6	0.8	0.056	3.00
0.969	0.05	18.69	2.6	0.8	0.075	3.00
0.131	0.01	4.43	2.6	0.8	0.035	1.40
0.336	0.03	5.49	2.6	0.8	0.055	1.40
0.179	0.01	5.63	2.6	0.8	0.039	1.60
0.456	0.04	7.01	2.6	0.8	0.062	1.60
0.225	0.01	9.93	2.6	0.8	0.039	2.40
0.568	0.03	12.42	2.6	0.8	0.060	2.40
0.291	0.01	11.66	2.6	0.8	0.042	2.60
0.733	0.04	14.62	2.6	0.8	0.067	2.60
0.652	0.05	10.12	2.6	0.8	0.070	2.00

Table 26 – Adopted data parameters for the additional best-guess data points

Location	d_{90} (m)	Manning's roughness	Local velocity (m/s)	Unit discharge	K1	K2
Theoretical	0.148	0.031	2.52	2.52	1.00	1.00
Theoretical	0.388	0.047	3.04	3.04	1.00	1.00
Theoretical	0.298	0.035	3.56	7.11	1.00	1.00
Theoretical	0.774	0.052	4.30	8.60	1.00	1.00
Theoretical	0.792	0.065	3.89	4.67	1.00	1.00
Theoretical	0.446	0.037	4.36	13.07	1.00	1.00
Theoretical	1.162	0.056	5.27	15.80	1.00	1.00
Theoretical	1.938	0.075	6.23	18.69	1.00	1.00
Theoretical	0.262	0.035	3.16	4.43	1.00	1.00
Theoretical	0.672	0.055	3.92	5.49	1.00	1.00
Theoretical	0.358	0.039	3.52	5.63	1.00	1.00
Theoretical	0.912	0.062	4.38	7.01	1.00	1.00
Theoretical	0.450	0.039	4.14	9.93	1.00	1.00
Theoretical	1.136	0.060	5.18	12.42	1.00	1.00
Theoretical	0.582	0.042	4.49	11.66	1.00	1.00
Theoretical	1.466	0.067	5.62	14.62	1.00	1.00
Theoretical	1.304	0.070	5.06	10.12	1.00	1.00

A best-fit analysis of the combined field, laboratory and best-guess deepwater data produced three equations. Equation 172 is a best-fit equation based on the same format as that presented by Abt and Johnson (1991).

(Uniform flow) $$d_{50,critical} = \frac{0.98\,(SF).K_1.K_2.S_o^{0.47}.q^{0.64}}{(s_r - 1)} \quad (172)$$

Equation 173 represents the best-fit equation with the incorporation of a flow depth factor in line with the work of Maynord (1978, 1988) and Maynord, Ruff & Abt (1989).

(Uniform flow) $$d_{50,critical} = \frac{111(SF).K_1.K_2.S_o^{0.47}.q^{0.48}.y^{0.2}}{(s_r - 1)} \quad (173)$$

Equation 174 is a modification of Equation 173 that was found to produce slightly better fit for those flow conditions that would result in a large rock size, i.e. Equation 174 produced a better fit for the more critical high stress flow conditions.

(Uniform flow) $$d_{50,critical} = \frac{127\,(SF).K_1.K_2.S_o^{0.5}.q^{0.5}.y^{0.25}}{(s_r - 1)} \quad (174)$$

The critical (i.e. SF = 1.0) d_{50} rock sizes determined by Equations 172 to 174 in comparison with the observed values are presented in Tables 27 and 28.

Table 27 – Critical d_{50} rock size determined by the best-fit equations for the best-guess, deepwater data set

Location	Unit discharge	Flow depth (m)	Observed d_{50} (m)	Eqn 172 d_{50} (m)	Eqn 173 d_{50} (m)	Eqn 174 d_{50} (m)
Theoretical	2.52	1.00	**74**	100	98	98
Theoretical	3.04	1.00	**194**	198	188	196
Theoretical	7.11	2.00	**149**	194	185	195
Theoretical	8.60	2.00	**387**	386	356	392
Theoretical	4.67	1.20	**396**	401	369	401
Theoretical	13.07	3.00	**223**	286	268	293
Theoretical	15.80	3.00	**581**	569	517	587
Theoretical	18.69	3.00	**969**	975	862	1010
Theoretical	4.43	1.40	**131**	164	157	162
Theoretical	5.49	1.40	**336**	350	323	351
Theoretical	5.63	1.60	**179**	212	201	212
Theoretical	7.01	1.60	**456**	469	428	473
Theoretical	9.93	2.40	**225**	275	257	279
Theoretical	12.42	2.40	**568**	591	533	603
Theoretical	11.66	2.60	**291**	339	314	344
Theoretical	14.62	2.60	**733**	751	670	771
Theoretical	10.12	2.00	**652**	659	592	671

Table 28 – Critical d_{50} rock size determined by the best-fit equations for the reduced field and laboratory data set

Location	Unit discharge	Flow depth (m)	Observed d_{50} (m)	Eqn 172 d_{50} (m)	Eqn 173 d_{50} (m)	Eqn 174 d_{50} (m)
Cherry Ck	1.49	0.53	**350**	317	297	313
Liberty Hill	1.96	0.65	**600**	464	434	471
Little Dry Ck	2.34	0.67	**450**	480	438	475
Lone Tree	2.74	0.81	**380**	370	343	368
Georgetown	3.02	0.79	**600**	612	555	613
Englewood	4.38	0.98	**450**	593	528	584
Gay St	5.00	1.29	**500**	480	443	489
Sandstone	0.12	0.07	**76**	86	82	75
Sandstone	0.08	0.06	**76**	84	80	73
Sandstone	0.05	0.04	**76**	72	70	63
Sandstone	0.23	0.11	**109**	125	116	110
Sandstone	0.16	0.09	**109**	117	110	103
Sandstone	0.16	0.08	**109**	134	124	117
Basalt	0.19	0.10	**94**	93	87	81
Basalt	0.18	0.09	**94**	109	99	94
Basalt	0.16	0.08	**94**	115	104	99
Angular	0.17	0.19	**104**	89	97	94
Angular	0.33	0.26	**130**	137	142	141
Angular	0.41	0.30	**157**	158	162	164
Angular	0.05	0.09	**56**	37	42	38
Rounded	0.09	0.17	**102**	80	93	89
Rounded	0.09	0.17	**102**	80	93	89
Angular	0.08	0.14	**51**	38	44	40
Angular	0.09	0.15	**51**	42	48	44
Angular	0.10	0.16	**51**	45	51	47
Rounded	0.19	0.24	**102**	102	114	109
Rounded	0.19	0.24	**102**	105	117	113
Angular	0.33	0.35	**102**	98	108	107
Angular	0.35	0.36	**102**	103	113	112
Rounded	0.06	0.21	**51**	45	58	54
Angular	0.38	0.37	**102**	114	124	124
Angular	0.42	0.24	**56**	52	51	47
Angular	0.10	0.09	**26**	21	21	18
Angular	0.14	0.20	**26**	18	21	18
Angular	0.03	0.06	**26**	22	24	21
Angular	0.03	0.06	**26**	21	24	21
Angular	0.03	0.06	**26**	20	23	19
Angular	0.04	0.07	**26**	24	27	23
Angular	0.10	0.15	**56**	45	50	47
Angular	0.12	0.15	**56**	49	53	50
Angular	0.12	0.15	**56**	49	53	50
Angular	0.17	0.19	**56**	60	65	61

After considering various issues and the primary purpose of this study, it was concluded that Equation 174 represented the best overall design equation.

Graphical representations of the output from Equation 174 in comparison with the observed critical rock size are presented in Figures 7 to 10.

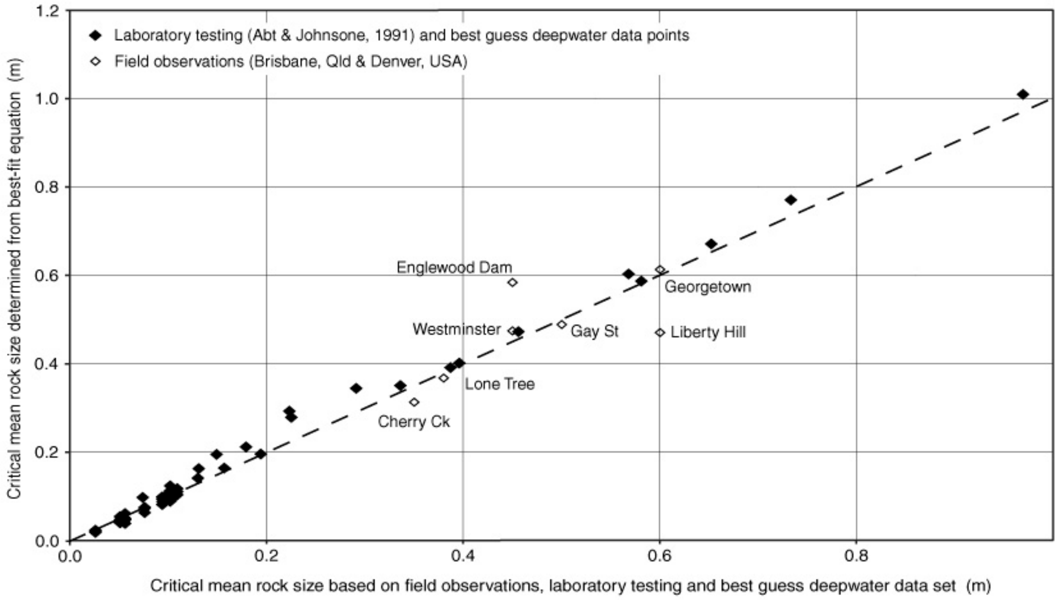


Figure 7 – Equation 174 output vs original data set (critical rock size, SF = 1.0)

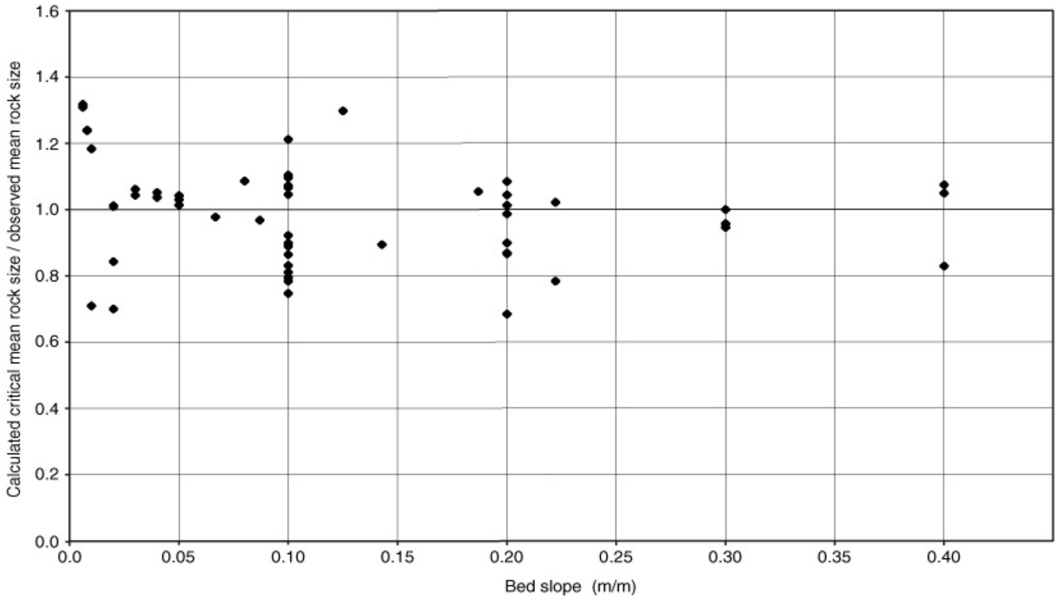


Figure 8 – Equation 174: ratio of output/observed vs bed slope, based on the critical rock size (SF = 1.0)

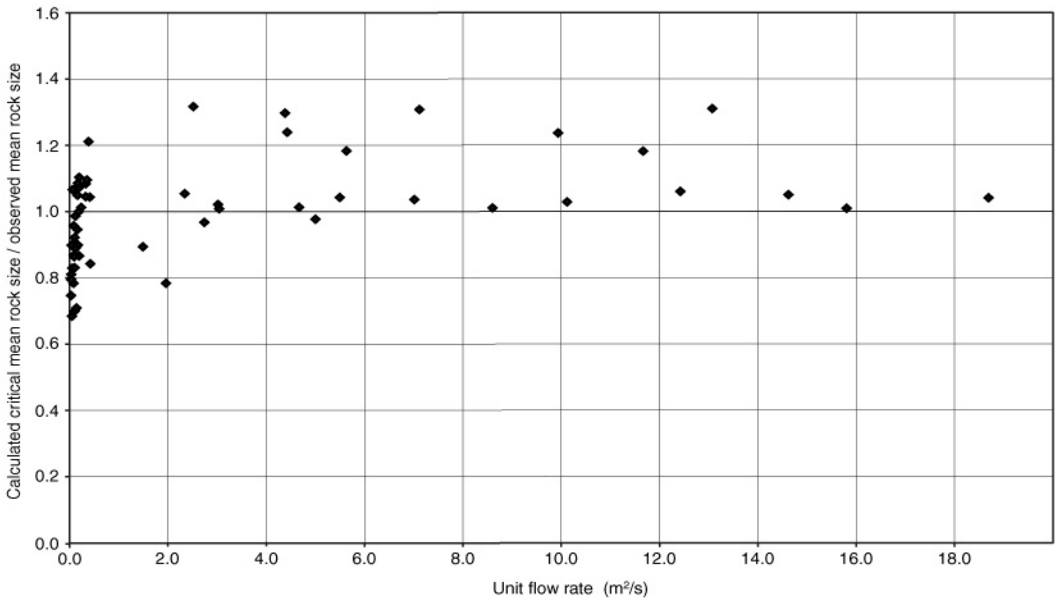


Figure 9 – Equation 174: ratio of output/observed vs unit flow rate, based on the critical rock size (SF = 1.0)

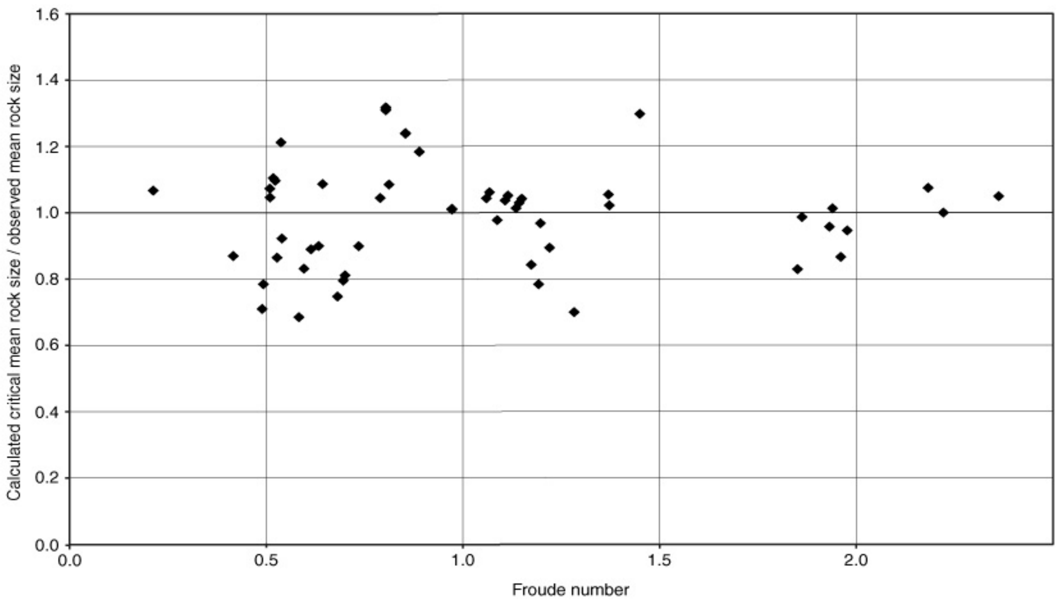


Figure 10 – Equation 174: ratio of output/observed vs Froude number, based on the critical rock size (SF = 1.0)

The results shown in Figures 8 to 10 indicate that a safety factor (SF) of 1.2 would in most cases provide a satisfactory design outcome resulting in a low risk of the design rock size being less than the actual critical rock size.

7. Development of various simplified design equations

To assist designers in the determination of a suitable rock size in specific design circumstances, various simplified design equations were developed from Equation 174. These equations rely on different input data and each equation has been assessed to determine the applicable range of bed slopes.

A simplified, uniform flow equation that is independent of flow depth and suitable for bed slopes, S_o < 50%.

$$d_{50} = \frac{SF.K_1.K_2.S_o^{0.47}.q^{0.64}}{(s_r - 1)} \tag{175}$$

A simplified, uniform flow equation based on velocity and bed slope, and suitable for bed slopes, S_o < 33%.

$$d_{50} = \frac{SF.K_1.K_2.V^2}{(A - B.\ln(S_o)).(s_r - 1)} \tag{176}$$

For SF = 1.2: A = 3.95, B = 4.97

For SF = 1.5: A = 2.44, B = 4.60

A simplified, uniform flow equation based on velocity and flow depth, and suitable for bed slopes, S_o < 10%.

$$d_{50} \approx \frac{K_1.V^{3.9}}{C.y^{0.95}(s_r - 1)} \tag{177}$$

where: C = 120 and 68 for SF = 1.2 and 1.5 respectively

A simplified, non-uniform flow equation suitable for use in the design of partially drowned waterway chutes with a bed slope, S_o < 50%.

$$d_{50} = \frac{127.SF.K_1.K_2.S_o^{0.5}.V^{2.5}.y^{0.75}}{V_o^{2.0}(s_r - 1)} \tag{178}$$

A simplified, non-uniform flow, velocity-based equation suitable for use in non-critical conditions only, and where the bed slope, S_o < 5%.

$$d_{50} \approx \frac{K_1.V^2}{2.g.K^2(s_r - 1)} \tag{179}$$

where: K =1.1 for low-turbulent deepwater flow, 1.0 for low-turbulent shallow water flow, and 0.86 for highly turbulent and/or supercritical flow.

Equations 176 to 179 are based on Manning's 'n' roughness being determined from Eqn 36. Equation 179 represents a modification of the equation originally presented by Isbash (1936); specifically, the coefficient for low turbulent flow is reduced from 1.2 to 1.1 (producing a slightly larger rock size) and a factor (K_1) is introduced to adjust for angular or rounded rock.

Table 9 provides the final recommended design equations for sizing rock on the bed of channels and chutes. The same equations can be used for sizing rock on bank slopes equal to or less than 2:1, but a 25% increase in rock size should be applied for bank slopes of 1.5:1.

Tables 29 to 35 provide design, mean rock size (rounded up to the next 0.1 m unit) for a safety factor of 1.2 and 1.5, based on Equation 176 for S_o = 0.1% and Equation 174 for all other bed slopes.

Table 29a – Preferred design table for mean rock size, d_{50} (m) for SF = 1.2

Safety factor, SF = 1.2			Specific gravity, s_r = 2.4			Size distribution, d_{50}/d_{90} = 0.5		
Unit flow rate (m^3/s/m)	**Bed slope (%)**							
	0.2	**0.5**	**1.0**	**2.0**	**3.0**	**4.0**	**5.0**	**6.0**
0.2	0.05	0.05	0.05	0.05	0.10	0.10	0.10	0.10
0.4	0.05	0.05	0.10	0.10	0.10	0.10	0.20	0.20
0.6	0.05	0.05	0.10	0.10	0.20	0.20	0.20	0.20
0.8	0.05	0.10	0.10	0.20	0.20	0.20	0.20	0.20
1.0	0.05	0.10	0.10	0.20	0.20	0.20	0.30	0.30
2.0	0.10	0.20	0.20	0.30	0.30	0.30	0.40	0.40
3.0	0.10	0.20	0.20	0.30	0.40	0.40	0.50	0.50
4.0	0.20	0.20	0.30	0.40	0.40	0.50	0.60	0.60
5.0	0.20	0.30	0.30	0.40	0.50	0.60	0.60	0.70
6.0	0.20	0.30	0.40	0.50	0.60	0.70	0.70	0.80
7.0	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
8.0	0.20	0.30	0.40	0.60	0.70	0.80	0.90	0.90
9.0	0.20	0.30	0.50	0.60	0.70	0.80	0.90	1.00
10.0	0.30	0.40	0.50	0.70	0.80	0.90	1.00	1.10
15.0	0.30	0.50	0.60	0.80	1.00	1.20	1.30	1.40
20.0	0.40	0.60	0.70	1.00	1.20	1.40	1.50	1.70

Table 29b – Preferred design table for mean rock size, d_{50} (m) for SF = 1.2

Safety factor, SF = 1.2			Specific gravity, s_r = 2.4			Size distribution, d_{50}/d_{90} = 0.5		
Unit flow rate (m^3/s/m)	**Bed slope (%)**							
	8.0	**9.0**	**10.0**	**15.0**	**20.0**	**25.0**	**33.3**	**50.0**
0.2	0.10	0.10	0.10	0.20	0.20	0.20	0.20	0.30
0.4	0.20	0.20	0.20	0.20	0.30	0.30	0.30	0.40
0.6	0.20	0.20	0.30	0.30	0.30	0.40	0.40	0.50
0.8	0.30	0.30	0.30	0.30	0.40	0.40	0.50	0.60
1.0	0.30	0.30	0.30	0.40	0.40	0.50	0.60	0.70
2.0	0.50	0.50	0.50	0.60	0.70	0.70	0.90	1.00
3.0	0.60	0.60	0.60	0.80	0.90	1.00	1.10	1.30
4.0	0.70	0.70	0.80	0.90	1.00	1.20	1.30	1.60
5.0	0.80	0.80	0.90	1.10	1.20	1.30	1.50	1.80
6.0	0.90	0.90	1.00	1.20	1.40	1.50	1.70	2.10
7.0	1.00	1.00	1.10	1.30	1.50	1.70	1.90	
8.0	1.10	1.10	1.20	1.40	1.60	1.80	2.10	
9.0	1.10	1.20	1.30	1.50	1.80	1.90	2.20	
10.0	1.20	1.30	1.40	1.60	1.90	2.10		
15.0	1.60	1.70	1.80	2.10	2.40			
20.0	1.90	2.00	2.10					

Table 30a – Alternative design table for mean rock size, d_{50} (m) for SF = 1.2

Safety factor, SF = 1.2			Specific gravity, s_r = 2.4			Size distribution, d_{50}/d_{90} = 0.5		
Local velocity (m/s)	**Bed slope (%)**							
	0.1	**0.5**	**1.0**	**2.0**	**3.0**	**4.0**	**5.0**	**6.0**
0.5	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
0.8	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
1.0	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
1.3	0.05	0.05	0.05	0.10	0.10	0.10	0.10	0.10
1.5	0.10	0.10	0.10	0.10	0.10	0.10	0.20	0.20
1.8	0.10	0.10	0.10	0.20	0.20	0.20	0.20	0.20
2.0	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20
2.3	0.20	0.20	0.20	0.20	0.30	0.30	0.30	0.30
2.5	0.20	0.20	0.20	0.30	0.30	0.30	0.30	0.40
2.8	0.20	0.30	0.30	0.30	0.40	0.40	0.40	0.40
3.0	0.30	0.30	0.30	0.40	0.40	0.40	0.50	0.50
3.5	0.40	0.40	0.40	0.50	0.50	0.60	0.60	0.70
4.0	0.50	0.50	0.50	0.60	0.70	0.80	0.80	0.80
4.5	0.60	0.60	0.70	0.80	0.90	0.90	1.00	1.10
5.0	0.70	0.70	0.80	1.00	1.10	1.10	1.20	1.30
6.0	1.00	1.00	1.20	1.40	1.50	1.60	1.70	1.80

Table 30b – Alternative design table for mean rock size, d_{50} (m) for SF = 1.2

Safety factor, SF = 1.2			Specific gravity, s_r = 2.4			Size distribution, d_{50}/d_{90} = 0.5		
Local velocity (m/s)	**Bed slope (%)**							
	8.0	**9.0**	**10.0**	**15.0**	**20.0**	**25.0**	**33.3**	**50.0**
0.5	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
0.8	0.05	0.05	0.05	0.05	0.05	0.05	0.10	0.10
1.0	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.20
1.3	0.10	0.10	0.10	0.20	0.20	0.20	0.20	0.20
1.5	0.20	0.20	0.20	0.20	0.20	0.20	0.30	0.30
1.8	0.20	0.20	0.20	0.30	0.30	0.30	0.30	0.40
2.0	0.30	0.30	0.30	0.30	0.30	0.40	0.40	0.50
2.3	0.30	0.30	0.30	0.40	0.40	0.50	0.50	0.60
2.5	0.40	0.40	0.40	0.50	0.50	0.60	0.60	0.70
2.8	0.50	0.50	0.50	0.60	0.60	0.70	0.70	0.80
3.0	0.50	0.60	0.60	0.70	0.70	0.80	0.90	1.00
3.5	0.70	0.70	0.80	0.90	1.00	1.00	1.10	1.30
4.0	0.90	1.00	1.00	1.10	1.20	1.30	1.50	1.70
4.5	1.10	1.20	1.20	1.40	1.50	1.70	1.90	2.10
5.0	1.40	1.50	1.50	1.70	1.90	2.00	2.20	
6.0	1.90	2.00	2.10	2.40	2.70			

Table 31a – Alternative design table for mean rock size, d_{50} (m) for SF = 1.2

Safety factor, SF = 1.2			Specific gravity, s_r = 2.4			Size distribution, d_{50}/d_{90} = 0.5		
Local flow depth (m)	**Bed slope (%)**							
	0.1	**0.5**	**1.0**	**2.0**	**3.0**	**4.0**	**5.0**	**6.0**
0.1	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
0.2	0.05	0.05	0.05	0.10	0.10	0.10	0.10	0.10
0.4	0.05	0.05	0.10	0.20	0.20	0.20	0.20	0.20
0.6	0.05	0.10	0.10	0.20	0.20	0.30	0.30	0.30
0.8	0.05	0.10	0.20	0.30	0.30	0.30	0.40	0.40
1.0	0.10	0.20	0.20	0.30	0.40	0.40	0.50	0.50
1.2	0.10	0.20	0.20	0.40	0.40	0.50	0.60	0.60
1.4	0.10	0.20	0.30	0.40	0.50	0.60	0.60	0.70
1.6	0.10	0.20	0.30	0.50	0.60	0.60	0.70	0.80
1.8	0.20	0.20	0.30	0.50	0.60	0.70	0.80	0.90
2.0	0.20	0.30	0.40	0.60	0.70	0.80	0.90	1.00
2.2	0.20	0.30	0.40	0.60	0.70	0.90	1.00	1.10
2.4	0.20	0.30	0.40	0.70	0.80	0.90	1.10	1.20
2.6	0.20	0.30	0.50	0.70	0.90	1.00	1.20	1.30
2.8	0.20	0.40	0.50	0.80	0.90	1.10	1.20	1.40
3.0	0.20	0.40	0.50	0.80	1.00	1.20	1.30	1.50

Table 31b – Alternative design table for mean rock size, d_{50} (m) for SF = 1.2

Safety factor, SF = 1.2			Specific gravity, s_r = 2.4			Size distribution, d_{50}/d_{90} = 0.5		
Local flow depth (m)	**Bed slope (%)**							
	8.0	**9.0**	**10.0**	**15.0**	**20.0**	**25.0**	**33.3**	**50.0**
0.1	0.10	0.10	0.10	0.10	0.10	0.20	0.20	0.20
0.2	0.20	0.20	0.20	0.20	0.20	0.30	0.30	0.40
0.4	0.30	0.30	0.30	0.40	0.40	0.50	0.50	0.70
0.6	0.40	0.40	0.40	0.50	0.60	0.70	0.80	1.00
0.8	0.50	0.50	0.60	0.70	0.80	0.90	1.00	1.30
1.0	0.60	0.60	0.70	0.80	1.00	1.10	1.30	1.60
1.2	0.70	0.80	0.80	1.00	1.20	1.30	1.50	1.90
1.4	0.80	0.90	0.90	1.10	1.30	1.50	1.70	2.20
1.6	0.90	1.00	1.10	1.30	1.50	1.70	2.00	
1.8	1.00	1.10	1.20	1.50	1.70	1.90	2.20	
2.0	1.20	1.20	1.30	1.60	1.90	2.10		
2.2	1.30	1.30	1.40	1.80	2.10			
2.4	1.40	1.50	1.60	1.90	2.30			
2.6	1.50	1.60	1.70	2.10				
2.8	1.60	1.70	1.80	2.20				
3.0	1.70	1.80	1.90	2.40				

Table 32a – Preferred design table for mean rock size, d_{50} (m) for SF = 1.5

Safety factor, SF = 1.5			Specific gravity, s_r = 2.4			Size distribution, d_{50}/d_{90} = 0.5	
Unit flow rate (m^3/s/m)	**Bed slope (%)**							

Unit flow rate (m^3/s/m)	0.1	0.5	1.0	2.0	3.0	4.0	5.0	6.0
0.2	0.05	0.05	0.05	0.10	0.10	0.10	0.10	0.10
0.4	0.05	0.05	0.10	0.10	0.20	0.20	0.20	0.20
0.6	0.05	0.10	0.10	0.20	0.20	0.20	0.20	0.30
0.8	0.05	0.10	0.20	0.20	0.20	0.20	0.30	0.30
1.0	0.10	0.10	0.20	0.20	0.30	0.30	0.30	0.30
2.0	0.10	0.20	0.20	0.30	0.40	0.40	0.50	0.50
3.0	0.20	0.20	0.30	0.40	0.50	0.50	0.60	0.60
4.0	0.20	0.30	0.40	0.50	0.60	0.60	0.70	0.80
5.0	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
6.0	0.20	0.30	0.40	0.60	0.70	0.80	0.90	1.00
7.0	0.30	0.40	0.50	0.70	0.80	0.90	1.00	1.10
8.0	0.30	0.40	0.50	0.70	0.90	1.00	1.10	1.20
9.0	0.30	0.40	0.60	0.80	0.90	1.10	1.20	1.30
10.0	0.30	0.50	0.60	0.80	1.00	1.10	1.20	1.40
15.0	0.40	0.60	0.80	1.10	1.30	1.50	1.60	1.80
20.0	0.50	0.70	0.90	1.30	1.50	1.80	1.90	2.10

Table 32b – Preferred design table for mean rock size, d_{50} (m) for SF = 1.5

Safety factor, SF = 1.5			Specific gravity, s_r = 2.4			Size distribution, d_{50}/d_{90} = 0.5	
Unit flow rate (m^3/s/m)	**Bed slope (%)**							

Unit flow rate (m^3/s/m)	8.0	9.0	10.0	15.0	20.0	25.0	33.3	50.0
0.2	0.20	0.20	0.20	0.20	0.20	0.20	0.30	0.30
0.4	0.20	0.20	0.20	0.30	0.30	0.40	0.40	0.50
0.6	0.30	0.30	0.30	0.40	0.40	0.40	0.50	0.60
0.8	0.30	0.30	0.40	0.40	0.50	0.50	0.60	0.70
1.0	0.40	0.40	0.40	0.50	0.60	0.60	0.70	0.80
2.0	0.60	0.60	0.60	0.70	0.80	0.90	1.10	1.30
3.0	0.70	0.80	0.80	1.00	1.10	1.20	1.40	1.70
4.0	0.90	0.90	1.00	1.20	1.30	1.50	1.70	2.00
5.0	1.00	1.00	1.10	1.30	1.50	1.70	1.90	2.30
6.0	1.10	1.20	1.20	1.50	1.70	1.90	2.20	
7.0	1.20	1.30	1.40	1.70	1.90	2.10		
8.0	1.30	1.40	1.50	1.80	2.10			
9.0	1.40	1.50	1.60	1.90	2.20			
10.0	1.50	1.60	1.70	2.10				
15.0	2.00	2.10	2.20					
20.0	2.40							

Table 33a – Alternative design table for mean rock size, d_{50} (m) for SF = 1.5

Safety factor, SF = 1.5			Specific gravity, s_r = 2.4			Size distribution, d_{50}/d_{90} = 0.5		
Local velocity (m/s)	**Bed slope (%)**							
	0.1	**0.5**	**1.0**	**2.0**	**3.0**	**4.0**	**5.0**	**6.0**
0.5	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
0.8	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
1.0	0.05	0.05	0.05	0.10	0.10	0.10	0.10	0.10
1.3	0.10	0.10	0.10	0.10	0.10	0.10	0.20	0.20
1.5	0.10	0.10	0.10	0.20	0.20	0.20	0.20	0.20
1.8	0.20	0.20	0.20	0.20	0.20	0.20	0.30	0.30
2.0	0.20	0.20	0.20	0.30	0.30	0.30	0.30	0.30
2.3	0.20	0.20	0.30	0.30	0.30	0.40	0.40	0.40
2.5	0.30	0.30	0.30	0.40	0.40	0.40	0.50	0.50
2.8	0.30	0.30	0.40	0.40	0.50	0.50	0.60	0.60
3.0	0.40	0.40	0.40	0.50	0.60	0.60	0.70	0.70
3.5	0.50	0.50	0.60	0.70	0.80	0.80	0.90	0.90
4.0	0.60	0.70	0.80	0.90	1.00	1.10	1.10	1.20
4.5	0.80	0.80	0.90	1.10	1.20	1.30	1.40	1.50
5.0	0.90	1.00	1.10	1.40	1.50	1.60	1.80	1.90
6.0	1.30	1.40	1.60	1.90	2.10	2.30	2.50	2.60

Table 33b – Alternative design table for mean rock size, d_{50} (m) for SF = 1.5

Safety factor, SF = 1.5			Specific gravity, s_r = 2.4			Size distribution, d_{50}/d_{90} = 0.5		
Local velocity (m/s)	**Bed slope (%)**							
	8.0	**9.0**	**10.0**	**15.0**	**20.0**	**25.0**	**33.3**	**50.0**
0.5	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
0.8	0.05	0.05	0.05	0.10	0.10	0.10	0.10	0.10
1.0	0.10	0.10	0.10	0.10	0.20	0.20	0.20	0.20
1.3	0.20	0.20	0.20	0.20	0.20	0.20	0.30	0.30
1.5	0.20	0.20	0.20	0.30	0.30	0.30	0.30	0.40
1.8	0.30	0.30	0.30	0.40	0.40	0.40	0.50	0.50
2.0	0.40	0.40	0.40	0.40	0.50	0.50	0.60	0.70
2.3	0.50	0.50	0.50	0.50	0.60	0.60	0.70	0.80
2.5	0.50	0.60	0.60	0.70	0.70	0.80	0.90	1.00
2.8	0.60	0.70	0.70	0.80	0.90	0.90	1.00	1.20
3.0	0.80	0.80	0.80	0.90	1.00	1.10	1.20	1.40
3.5	1.00	1.10	1.10	1.30	1.40	1.50	1.70	1.90
4.0	1.30	1.40	1.40	1.60	1.80	1.90	2.10	2.50
4.5	1.70	1.70	1.80	2.00	2.20	2.40		
5.0	2.00	2.10	2.10	2.50				
6.0	2.80							

Table 34a – Alternative design table for mean rock size, d_{50} (m) for SF = 1.5

Safety factor, SF = 1.5			Specific gravity, s_r = 2.4			Size distribution, d_{50}/d_{90} = 0.5		
Local flow depth (m)	**Bed slope (%)**							
	0.1	**0.5**	**1.0**	**2.0**	**3.0**	**4.0**	**5.0**	**6.0**
0.1	0.05	0.05	0.05	0.05	0.05	0.05	0.10	0.10
0.2	0.05	0.05	0.05	0.10	0.10	0.10	0.20	0.20
0.4	0.05	0.10	0.10	0.20	0.20	0.20	0.30	0.30
0.6	0.05	0.10	0.20	0.20	0.30	0.30	0.40	0.40
0.8	0.10	0.20	0.20	0.30	0.40	0.40	0.50	0.50
1.0	0.10	0.20	0.20	0.30	0.40	0.50	0.60	0.60
1.2	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.70
1.4	0.20	0.20	0.30	0.50	0.60	0.70	0.80	0.80
1.6	0.20	0.30	0.40	0.50	0.70	0.80	0.90	0.90
1.8	0.20	0.30	0.40	0.60	0.70	0.80	1.00	1.00
2.0	0.20	0.30	0.40	0.60	0.80	0.90	1.10	1.20
2.2	0.20	0.30	0.50	0.70	0.90	1.00	1.20	1.30
2.4	0.20	0.40	0.50	0.80	1.00	1.10	1.30	1.40
2.6	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.50
2.8	0.30	0.40	0.60	0.90	1.10	1.30	1.50	1.60
3.0	0.30	0.40	0.60	0.90	1.20	1.40	1.60	1.70

Table 34b – Alternative design table for mean rock size, d_{50} (m) for SF = 1.5

Safety factor, SF = 1.5			Specific gravity, s_r = 2.4			Size distribution, d_{50}/d_{90} = 0.5		
Local flow depth (m)	**Bed slope (%)**							
	8.0	**9.0**	**10.0**	**15.0**	**20.0**	**25.0**	**33.3**	**50.0**
0.1	0.10	0.10	0.10	0.10	0.20	0.20	0.20	0.20
0.2	0.20	0.20	0.20	0.20	0.30	0.30	0.30	0.40
0.4	0.30	0.30	0.30	0.40	0.50	0.50	0.60	0.80
0.6	0.40	0.50	0.50	0.60	0.70	0.80	0.90	1.10
0.8	0.60	0.60	0.60	0.80	0.90	1.00	1.20	1.50
1.0	0.70	0.70	0.80	1.00	1.10	1.30	1.50	1.80
1.2	0.80	0.90	0.90	1.20	1.30	1.50	1.80	2.20
1.4	1.00	1.00	1.10	1.30	1.60	1.80	2.00	
1.6	1.10	1.20	1.20	1.50	1.80	2.00		
1.8	1.20	1.30	1.40	1.70	2.00			
2.0	1.40	1.40	1.50	1.90	2.20			
2.2	1.50	1.60	1.70	2.10				
2.4	1.60	1.70	1.80	2.30				
2.6	1.70	1.90	2.00					
2.8	1.90	2.00						
3.0	2.00							

Table 35a – Alternative design table for mean rock size for specific gravity, s_r = 2.6

Safety factor, SF = 1.5			Specific gravity, s_r = 2.6			Size distribution, d_{50}/d_{90} = 0.5	
Local flow depth (m)	**Bed slope (%)**							

Local flow depth (m)	0.1	0.5	1.0	2.0	3.0	4.0	5.0	6.0
0.1	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.10
0.2	0.05	0.05	0.05	0.10	0.10	0.10	0.10	0.20
0.4	0.05	0.05	0.10	0.20	0.20	0.20	0.20	0.30
0.6	0.05	0.10	0.20	0.20	0.30	0.30	0.30	0.40
0.8	0.10	0.10	0.20	0.30	0.30	0.40	0.40	0.50
1.0	0.10	0.20	0.20	0.30	0.40	0.40	0.50	0.60
1.2	0.10	0.20	0.30	0.40	0.50	0.50	0.60	0.70
1.4	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80
1.6	0.20	0.20	0.30	0.50	0.60	0.70	0.80	0.90
1.8	0.20	0.30	0.40	0.50	0.70	0.80	0.90	1.00
2.0	0.20	0.30	0.40	0.60	0.70	0.80	1.00	1.10
2.2	0.20	0.30	0.40	0.60	0.80	0.90	1.00	1.20
2.4	0.20	0.30	0.50	0.70	0.90	1.00	1.10	1.30
2.6	0.20	0.40	0.50	0.70	0.90	1.10	1.20	1.40
2.8	0.20	0.40	0.50	0.80	1.00	1.20	1.30	1.50
3.0	0.20	0.40	0.60	0.90	1.10	1.20	1.40	1.60

Table 35b – Alternative design table for mean rock size for specific gravity, s_r = 2.6

Safety factor, SF = 1.5			Specific gravity, s_r = 2.6			Size distribution, d_{50}/d_{90} = 0.5	
Local flow depth (m)	**Bed slope (%)**							

Local flow depth (m)	8.0	9.0	10.0	15.0	20.0	25.0	33.3	50.0
0.1	0.10	0.10	0.10	0.10	0.10	0.20	0.20	0.20
0.2	0.20	0.20	0.20	0.20	0.20	0.30	0.30	0.40
0.4	0.30	0.30	0.30	0.40	0.40	0.50	0.60	0.70
0.6	0.40	0.40	0.50	0.60	0.60	0.70	0.80	1.00
0.8	0.50	0.60	0.60	0.70	0.80	0.90	1.10	1.30
1.0	0.60	0.70	0.70	0.90	1.00	1.20	1.30	1.70
1.2	0.80	0.80	0.90	1.10	1.20	1.40	1.60	2.00
1.4	0.90	0.90	1.00	1.20	1.40	1.60	1.90	2.30
1.6	1.00	1.10	1.10	1.40	1.60	1.80	2.10	
1.8	1.10	1.20	1.30	1.60	1.80	2.00		
2.0	1.20	1.30	1.40	1.70	2.00			
2.2	1.30	1.40	1.50	1.90	2.20			
2.4	1.50	1.60	1.70	2.10				
2.6	1.60	1.70	1.80	2.20				
2.8	1.70	1.80	1.90	2.40				
3.0	1.80	1.90	2.10					

8. Symbols and terminology

a = effective shear stress surface area of rock [m^2]

A = cross sectional area of channel [m^2]

A = equation constant that typically varies with the degree of turbulence, rock texture (e.g. rounded or fractured) and the required factor of safety

B = constant that typically varies with the degree of turbulence

C = coefficient (0.22 for gravel and pebbles, 0.27 for crushed stone)

C_u = coefficient of uniformity in relation to the distribution of rock size = d_{60}/d_{10}

d = nominal rock diameter [m]

d_x = nominal rock size (diameter) of which X% of the rocks are smaller [m]

$d_{X,critical}$ = critical rock diameter for which movement is just expected [m]

$d_{50,critical}$ = mean rock diameter for which movement is expected to commence [m]

D_o = internal diameter of outlet pipe [m]

F = resultant force on rock [N]

F = Froude number based on flow depth (y) rather than hydraulic radius [dimensionless]

F^* = Shield's parameter (variable depending on Reynolds number and rock size, d_x)

F_d = drag force of rock [N]

F_o = Froude number of flow at the pipe exit [dimensionless]

F_R = resisting force on rock [N]

F_w = weight component of water cell parallel to channel bed [N]

g = acceleration due to gravity [m/s^2]

k = roughness coefficient (Strickler's coefficient)

K = equation constant based on flow conditions (degree of local turbulence)

K_1 = correction factor for rock shape

K_2 = correction factor for rock grading

L = length of water cell or control volume [m]

n = Manning's roughness value [dimensionless]

n_o = Manning's roughness value for deepwater conditions [dimensionless]

n_p = porosity of rock embankment

P = wetted perimeter [m]

q = flow per unit width down the embankment [$m^3/s/m$]

Q = total discharge [m^3/s]

R = hydraulic radius ($R = A/P$) [m]

s_r = specific gravity of rock

S = slope [m/m]

S_e = slope of energy line [m/m]

S_o = bed slope [m/m]

SF = factor of safety as applied to rock size (d)

SF_F = factor of safety as applied to the net displacement force of the rock (F)

= the ratio of the moments of forces resisting rotation of the rock particle out of the riprap blanket to the moments tending to dislodge the particle out of the riprap layer into the flow

SF_V = factor of safety as applied to the design flow velocity (V)

SF_W = factor of safety as applied to rock weight (W)

TW = Tailwater depth relative to pipe invert [m]

V = average flow velocity [m/s]

= local flow velocity average over depth [m/s]

V_{avg} = the average channel velocity

V_{bend} = the maximum depth-average velocity in the bend over the toe of the side slope

W = weight of water cell [N]

W = weight of d_{33} rock of which 67% of rock is larger [kg]

y = depth of flow to bed at a given location [m]

α = bank slope relative to the channel bed [degrees]

β = bank slope relative to the horizontal [degrees]

θ = slope of channel bed [degrees]

ρ_w = density of water [kg/m^3]

τ_c = critical shear stress at which point rocks begin to move [N/m^2]

τ_o = average shear stress on the surface (bed and banks) of the channel [Pa = N/m^2]

Φ = angle of repose of rock [degrees]

φ = local gradient of energy line [degrees]

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