



Catchments
& Creeks

Catchments & Creeks Pty. Ltd.

ABN 34 078 164 119

PO Box 314
FERNY HILLS, QLD 4055
(W/H) 07 3351 3505
FAX 07 3351 3536
Mobile: 0417 733 396

3 November 2009

REPORT ON THE SELECTION OF ROCK-SIZING EQUATIONS FOR USE IN THE DESIGN OF ROCK-LINED CHANNELS, CHUTES AND SPILLWAYS

1.0 Introduction

The purpose of report is to:

- (i) outline the logic used in selecting the rock-sizing equations used within the erosion and sediment control fact sheets prepared as part of the IECA Best Practice erosion and Sediment Control (2008) document;
- (ii) compare the various rock sizing procedures based on allowable shear stress and allowable flow velocity;
- (iii) resolve the apparent conflict between the various reported procedures for sizing rock on the beds and banks of low gradient channels and steep chutes (spillways);
- (iv) determine an appropriate procedure for adjusting the rock size in circumstance where either white water conditions exist, or the average flow depth is less than the mean rock size.

2.0 Angle of repose of rock

The angle of repose of rock is used in a number of rock-sizing equations presented for the design of steep-gradient chutes and spillways.

Chow (1959) suggested angles of repose for rock as per Table 1.

Table 1 – Angle of repose for rock (100–500mm)

Rock condition	Angle of repose, Φ (degrees)
Very angular rock	41°
Slightly angular rock	40°
Moderately rounded rock	39°
Rounded rock	37–38°

Henderson (1966) (after Simons & Albertson, 1960) suggested angles of repose for rock as per Table 2.

Table 2 – Angle of repose for large rock (> 500mm)

Rock condition	Angle of repose, Φ (degrees)
Very angular rock	42°
Very rounded rock	40°

3.0 Density of rock

Rock density appears to be a major factor in assessing its stability while subject to water flow.

The Standing Committee on Rivers and Catchments, Victoria (1991) suggests the following rock densities:

Table 3 – Relative density (specific gravity) of rock

Rock type	Relative density (s_r)
Sandstone	2.1 to 2.4
Granite	2.5 to 3.1 commonly 2.6
Limestone	2.6
Basalt	2.7 to 3.2

4.0 Factor of safety

Various authors have recommended various safety factors and various ways of incorporating a factor of safety into the sizing of rock as used in hydraulic structures.

In some cases a safety factor (SF) is applied directly to the rock size such that:

$$d_x = SF \cdot d_{x,critical} \quad (1)$$

where:

d_x = design nominal rock diameter of which X% of the rocks are smaller [m]

$d_{x,critical}$ = critical rock diameter for which movement is just expected [m]

A factor of safety (SF) applied directly to the rock size may be compared to an equivalent factor of safety applied to the design flow velocity (SF_V), or rock weight (SF_W), using the following equations.

$$SF_V = SF^{1/2} \quad (2)$$

$$SF_W = SF^3 \quad (3)$$

The above equations are based on the following assumptions:

$$d_{50} = \text{constant} (V^2) \quad (4)$$

$$\text{Rock weight} (W_s) = (\text{volume of rock}) \cdot g \cdot \rho_w \cdot s_r \quad (5)$$

$$\text{Volume of rock} = \text{constant} (d^3) \quad (6)$$

A comparison between the various types of safety factors is presented in Table 4.

Table 4 – Comparison of various safety factors

Safety factor relative to rock size (SF)	1.20	1.50	1.80	2.0
Safety factor relative to flow velocity (SF _v)	1.10	1.22	1.34	1.41
Safety factor relative to weight of rock (SF _w)	1.73	3.88	5.83	8.00

5.0 Manning’s roughness of rock-lined surfaces

Strickler’s equation has been used for some time to estimate of the Manning’s n roughness for rock lined channels. However, this equation should only be considered appropriate for deepwater conditions where the flow depth is significantly greater than the rock size.

The Strickler equation for fluid motion (Equation 7) is similar to Manning’s equation, except a coefficient (k) is used instead of Manning’s roughness coefficient.

$$V = k R^{2/3} S^{1/2} \quad (7)$$

where: V = mean flow velocity [m/s]
 k = roughness coefficient (Strickler’s coefficient)
 R = hydraulic radius [m]
 S = bed slope (uniform flow) [m/m]

The roughness equation developed by Strickler is presented below (Equation 8).

$$k = 21.1/(d_{50})^{1/6} \quad (8)$$

Strickler’s work allowed the development of an equivalent Manning’s roughness formula.

(metric form)
$$n_o = ((d_{50})^{1/6})/21.1 \quad (9)$$

where: n_o = Manning’s roughness coefficient (assumed to be deepwater condition)
 d₅₀ = mean rock size for which 50% of rocks are smaller [m]

Strickler’s work is often confused with formula presented by Meyer-Peter & Muller (Equation 10).

(metric form)
$$n_o = ((d_{90})^{1/6})/26.0 \quad (10)$$

Analysing stream gauging data from New Zealand (Wetheridge 2002) an equation was developed for the Manning’s roughness coefficient (n) for rock-lined channels in deep and shallow water. In deepwater conditions this equation is comparable with the Meyer-Peter & Muller equation.

(metric from)
$$n = \frac{(d_{90})^{1/6}}{26(1 - 0.3593^{(x)^{0.7}})} \quad (11)$$

where: X = (R/d₉₀)(d₅₀/d₉₀)
 d₉₀ = mean rock size for which 90% of rocks are smaller [m]

For ‘natural’ rock extracted from streambeds the relative roughness value (d₅₀/d₉₀) is typically in the range 0.2 to 0.5. For quarried rock the ratio is more likely to be in the range 0.5 to 0.8. Table 5 contains the data range for those data point where the calculated Manning’s n value (using Equation 11) falls within ± 10% of the observed Manning’s n value.

Table 5 – Data range used in determination of Equation 11

Criteria	d50 (mm)	d90 (mm)	R/d50	R/d90	n _o /n	d50/d90
Min (+/-10%)	16	90	2.31	0.73	0.284	0.080
Max (+/-10%)	112	350	55.6	12.0	1.080	0.661
Min (all data)	16	90	1.17	0.31	0.097	0.080
Max (all data)	397	1080	66.9	12.9	1.120	0.661

6.0 Shear stress equation

The gravitational forces acting on a body of water are shown in Figure 1.

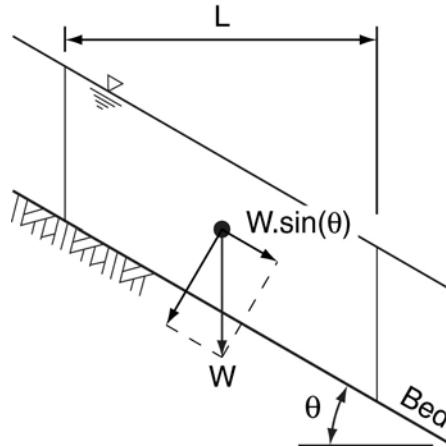


Figure 1 – Gravitational forces of body of water

For a control volume of water, the component of the weight tangential to the channel bed (F_w) is given by Equation 1.

$$F_w = W \cdot \sin(\theta) \quad (12)$$

where:

$$W = \rho_w \cdot g \cdot A \cdot L \quad (13)$$

In uniform flow conditions there is no rapid acceleration of the water, thus the weight component must be balanced by the tractive force exerted by the channel bed on the water, therefore:

$$\tau_o \cdot P \cdot L = W \cdot \sin(\theta) = \rho_w \cdot g \cdot A \cdot L \cdot \sin(\theta) \quad (14)$$

where:

F_w = weight component of water cell parallel to channel bed [N]

W = weight of water cell [N]

θ = slope of channel bed [degrees]

P = wetted perimeter [m]

ρ_w = density of water [kg/m³]

g = acceleration due to gravity [m/s²]

A = cross sectional area of channel [m²]

L = length of water cell [m]

τ_o = average shear stress on the surface (bed and banks) of the channel [Pa = N/m²]

Therefore the average shear stress may be given by:

$$\tau_o = \rho_w \cdot g \cdot R \cdot \sin(\theta) \quad (15)$$

Now during uniform flow on a low-gradient channel, the energy slope has the following properties:

$$S_e = S_o = \theta = \tan(\theta) \approx \sin(\theta) \quad (16)$$

Thus:

$$\tau_o = \rho_w \cdot g \cdot R_h \cdot S_o \quad (17)$$

where:

R = hydraulic radius (R = A/P) [m]

S_e = slope of energy line [m/m]

S_o = channel bed slope [m/m]

Equation 15 represents the **average** shear stress on the bed and banks; however, the actual shear stress varies across the bed and banks. Chow (p.170) indicates that the maximum shear stress on the bed and banks for a **trapezoidal** channel may be approximated by Equations 18 and 19.

Maximum shear stress on bed:
$$\tau_{max} = \rho_w \cdot g \cdot y \cdot \sin(\theta) \quad (18)$$

Maximum shear stress on banks:
$$\tau_{max} = (0.75) \cdot \rho_w \cdot g \cdot y \cdot \sin(\theta) \quad (19)$$

where:

y = depth of flow across the channel bed [m]

The above equations are all based on uniform flow conditions. In cases where the energy slope (S_e) is significantly different from the bed slope (S_o), the water will experience either acceleration or deceleration. Under those conditions where the water is accelerating, the weight component in the direction of flow (W·sin(θ)) will be greater than the effective shear stress, i.e. there is a resulting gravitational force acting on the water causing it to accelerate. In such cases, the effective shear stress is assumed to be proportional to the energy slope rather than the bed slope.

Thus, for **non-uniform** flow on both steep and low-gradient trapezoidal channels, the following conditions apply:

Average shear stress on channel:
$$\tau_o = \rho_w \cdot g \cdot R \cdot \sin(\varphi) \quad (20)$$

Maximum shear stress on bed:
$$\tau_{max} = \rho_w \cdot g \cdot y \cdot \sin(\varphi) \quad (21)$$

Maximum shear stress on banks:
$$\tau_{max} = (0.75) \cdot \rho_w \cdot g \cdot y \cdot \sin(\varphi) \quad (22)$$

where:

φ = local gradient of the energy line [degrees]

S_e = tan(φ) = slope of energy line [m/m]

and for non-uniform flow:
$$\sin(\varphi) \neq \sin(\theta) \quad (23)$$

Now:

$$\sin(\varphi) = \cos(\varphi) \cdot \tan(\varphi) = \cos(\varphi) \cdot S_e \quad (24)$$

Thus Equations 20 to 22 may also be written as:

$$\text{Average shear stress on channel: } \tau_o = \rho_w \cdot g \cdot R \cdot \cos(\varphi) \cdot S_e \quad (25)$$

$$\text{Maximum shear stress on bed: } \tau_{\max} = \rho_w \cdot g \cdot y \cdot \cos(\varphi) \cdot S_e \quad (26)$$

$$\text{Maximum shear stress on banks: } \tau_{\max} = (0.75) \cdot \rho_w \cdot g \cdot y \cdot \cos(\varphi) \cdot S_e \quad (27)$$

7.0 Shield's critical shear stress

The critical shear stress on the bed of a very low gradient channel is given by Equation 28.

$$\tau_c = F^* \cdot \rho_w \cdot g \cdot d_x \cdot (s_r - 1) \quad (28)$$

where:

τ_c = critical shear stress at which point rocks begin to move [N/m²]

F^* = Shield's parameter (variable depending on Reynolds number and rock size, d_x)

d_x = nominal rock size (diameter) of which X% of the rocks are smaller [m]

s_r = specific gravity of rock

The following section outlines the critical shear stress Shield's parameters presented by various authors.

7.1 Chow (1959) and Henderson (1966):

Critical shear stress based on d_{75} rock size:

$$\tau_c = 754 \cdot d_{75} \quad (29)$$

If the specific gravity of the rock (s_r) is assumed to be 2.6, gravity is 9.8m/s², and the water density is 1000kg/m³, then Equation 31 becomes:

$$\tau_c = (0.048) \rho_w \cdot g \cdot d_{75} (s_r - 1) \quad (30)$$

7.2 Meyer-Peter and Muller formula (reported in Standing Committee on Rivers and Catchments, Victoria, 1991):

For a flat bed the Meyer-Peter and Muller formula (1948) is presented as:

$$\tau_c = (0.047) \rho_w \cdot g \cdot d_{50} (s_r - 1) \quad (31)$$

It would initially appear (refer to Table 6) that Equation 31 would be more consistent with other reported critical shear stress equations if the rock size used in the equation was actually d_{75} or d_{90} instead of the reported d_{50} . However, this may actually reflect the variability of the critical shear stress for various flow conditions.

7.3 Newbury and Gaboury (1993):

Critical shear stress based on d_{50} rock size:

$$\tau_c = 980 \cdot d_{50} \quad (32)$$

If the specific gravity of the rock (s_r) is assumed to be 2.6, gravity is 9.8m/s^2 , and the water density is 1000kg/m^3 , then Equation 29 becomes:

$$\tau_c = (0.063)\rho_w \cdot g \cdot d_{50} (s_r - 1) \quad (33)$$

7.4 E. W. Lane (presented in L.W. Mays, 2005):

Critical shear stress developed for clear water flow based on d_{75} rock size:

$$\tau_c = 785 \cdot d_{75} \quad (34)$$

If the specific gravity of the rock (s_r) is assumed to be 2.6, gravity is 9.8m/s^2 , and the water density is 1000kg/m^3 , then Equation 34 becomes:

$$\tau_c = (0.050)\rho_w \cdot g \cdot d_{75} (s_r - 1) \quad (35)$$

7.5 Shields (presented in L.W. Mays, 2005):

Critical shear stress based on d_{50} rock size:

$$\tau_c = 990 \cdot d_{50} \quad (36)$$

If the specific gravity of the rock (s_r) is assumed to be 2.6, gravity is 9.8m/s^2 , and the water density is 1000kg/m^3 , then Equation 27 becomes:

$$\tau_c = (0.063)\rho_w \cdot g \cdot d_{50} (s_r - 1) \quad (37)$$

7.6 Application of the Shield's parameter and shear stress equation

From Equation 28 the **critical** rock size (d_{50}) on the **bed** of a **low-gradient channel** under **uniform flow conditions** is given by:

$$\tau_c = F^* \cdot \rho_w \cdot g \cdot d_x \cdot (s_r - 1) \quad (38)$$

incorporating Equation 17:

$$\tau_c = F^* \cdot \rho_w \cdot g \cdot d_x \cdot (s_r - 1) = \rho_w \cdot g \cdot R \cdot S_o \quad (39)$$

if F^* is based on d_{50} , then:

$$F^* \cdot d_{50} \cdot (s_r - 1) = R \cdot S_o \quad (40)$$

therefore;

$$d_{50, \text{critical}} = \frac{R \cdot S_o}{F^* (s_r - 1)} \quad (41)$$

where:

$d_{50, \text{critical}}$ = mean rock diameter for which movement is expected to commence [m]

A safety factor (SF) can be introduced to determine the **design** rock size:

$$\text{Uniform flow: } d_{50} = \frac{\text{SF} \cdot R \cdot S_o}{F^* (s_r - 1)} \quad (42)$$

A comparison can be made with the allowable velocity method (Section 8) where:

$$d_{50} = K \cdot V^2 \quad (43)$$

From Manning's equation:
$$V^2 = \frac{1}{n^2} R^{4/3} S_o \quad (44)$$

thus,

$$K = \frac{SF \cdot n^2}{F^* (s_r - 1) R^{1/3}} \quad (45)$$

where:

d_{50} = nominal rock diameter of which 50% of the rocks are smaller [m]

K = constant which varies with the degree of local turbulence

V = average flow velocity [m/s]

n = Manning's roughness value [dimensionless]

Suggested values are SF = 1.5, and F* = 0.07 (applicable when determining the d_{50} rock size – refer to Table 6).

Technical Note:

The above equations only apply to low-gradient channels because the potential effects of gravity contributing to the displacement of rocks down a slope have not been considered.

It should be noted that Maynard, Ruff & Abt (1989) report that investigations by Meyer-Peter & Muller 1948, Blench 1966, Neill 1967 and Bogardi 1978, have shown “that the Shield's coefficient is not constant as used in many riprap design procedures, but varies directly with relative roughness (particle size/depth)”. For further discussion, refer to Section 9.1.

Under **non-uniform flow conditions** on a low-gradient channel, Equation 42 becomes:

Non-uniform flow:
$$d_{50} = \frac{SF \cdot R \cdot \cos(\varphi) \cdot S_e}{F^* (s_r - 1)} \quad (46)$$

However, for a low-gradient channel the energy slope (φ) is likely to be small, thus:

$$\cos(\varphi) \approx 1.0 \quad (47)$$

and:

$$d_{50} = \frac{SF \cdot R \cdot S_e}{F^* (s_r - 1)} \quad (48)$$

A summary of the various recommended Shield's parameters is provided in Table 6.

Table 6 – Summary of Shield's parameter

Author	F* for d_{50}	F* for d_{75}
Meyer-Peter & Muller ^[1]	0.047	0.031 ^[3]
Meyer-Peter & Muller ^[2]	0.071 ^[3]	0.047
E.W. Lane	0.075 ^[3, 4]	0.050 ^[4]
Shields	0.063 ^[4]	0.042 ^[3, 4]
Newbury & Gaboury	0.063 ^[4]	0.042 ^[3, 4]
McLauchlin Engineers (refer to Section 11.1)	0.091	0.061 ^[3]
Chow & Henderson	0.072 ^[3, 4]	0.048 ^[4]

Notes: [1] Equations as presented in Standing Committee on Rivers and Catchments (1991).

[2] Meyer-Peter & Muller's Shield's parameter if assumed to be based on d_{75} rather than d_{50} .

[3] Based on $d_{75} = 1.5(d_{50})$ refer to Table 9.

[4] Based on specific gravity = 2.6, gravity = 9.8 m/s², and water density = 1000 kg/m³.

8.0 Allowable velocity method

The allowable velocity method for sizing rock is based on the development of a direct relationship between the required rock size and the local average flow velocity. These equations usually take the form of:

$$d_{50} = A.V^B \quad (49)$$

where:

A = constant that typically varies with the degree of turbulence, rock texture (e.g. rounded or fractured) and the required factor of safety

B = constant that typically varies with the degree of turbulence

Typically, these allowable flow velocity equations are only applicable for low-gradient channels (say less than 10%) operating under deepwater conditions (i.e. $y \gg d_{50}$) where there is only a minor gravity force component down the face of the slope, and whitewater flow conditions do not exist.

8.1 Isbash (1936)

Isbash investigated the stability of rounded rock dumped into flowing rivers for dam construction. His work focused on sizing individual stones located on the downstream dam slope to resist displacement due to overtopping flow and percolation through the embankment.

The results of Isbash (1936) are presented in McLaughlin Water Engineers (1986) and below as Equation 50.

$$d_{50} = \frac{V^2}{2.g.K^2.(s_r - 1)} \quad (50)$$

where:

K = equation constant based on flow conditions

Isbash recommended $K = 0.86$ for high turbulent river flow and $K = 1.2$ for low turbulent river flow, thus:

Low turbulent channel flow:
$$d_{50} = \frac{(0.035).V^2}{(s_r - 1)} \quad (51)$$

High turbulent channel flow:
$$d_{50} = \frac{(0.069).V^2}{(s_r - 1)} \quad (52)$$

Results of Isbash's equation are summarised in Table 7.

Table 7 – Rock size (m) for low-gradient channel bed based on Isbash's equation

Rock type	Assumed specific gravity	Low turbulent flow	High turbulent flow
Sandstone	2.2	$d_{50} = 0.030 V^2$	$d_{50} = 0.057 V^2$
Sandstone	2.4	$d_{50} = 0.025 V^2$	$d_{50} = 0.049 V^2$
Granite & Limestone	2.6	$d_{50} = 0.022 V^2$	$d_{50} = 0.043 V^2$
Basalt	2.9	$d_{50} = 0.019 V^2$	$d_{50} = 0.036 V^2$

8.2 California Department of Public Works (1960)

Equation 53 was presented within California Department of Public Works (1960) for deepwater, low-turbulent river flows.

$$W = \frac{1.132 \times 10^{-2} V^6 s_r}{(s_r - 1)^3 \cdot \sin^3(70^\circ - \alpha)} \quad (53)$$

where:

W = weight of d_{33} rock of which 67% of rock is larger [kg]

V = average flow velocity [m/s]

s_r = specific gravity of rock

70° = constant for randomly placed rock

α = bank slope relative to the channel bed [degrees]

This publication based its analysis on a coefficient of uniformity ($C_u = d_{60}/d_{10}$) of 1.10.

If spherical rocks are assumed then the nominal diameter may be determined from Equation 54.

$$d = \left(\frac{6 \cdot W}{\pi \cdot s_r \times 10^3} \right)^{1/3} \quad (54)$$

where:

d = nominal rock diameter [m]

Thus the d_{33} rock size may be determined from Equation 55.

$$d_{33} = \frac{0.0279 \cdot V^2}{(s_r - 1) \cdot \sin(70^\circ - \alpha)} \quad (55)$$

where:

d_{33} = nominal rock diameter of which 33% of the rocks are smaller [m]

For rock with a specific gravity (s_r) of 2.6 on a low-gradient channel bed (i.e. $\alpha = 0^\circ$) then:

$$d_{33} = 0.019 V^2 \quad (56)$$

Table 8 presents the equivalent rock-sizing equations for low-gradient channel beds under medium turbulence if the rock size distribution as presented in Table 6. These equations compare well with the rock sizing equations presented in Table 7.

Table 8 – Rock size (m) for low-gradient channel bed based on California Department of Public Works

Rock type	Specific gravity	Rock sizing equations	
Sandstone	2.2	$d_{33} = 0.025 V^2$	$d_{50} = 0.041 V^2$
Sandstone	2.4	$d_{33} = 0.021 V^2$	$d_{50} = 0.035 V^2$
Granite/Limestone	2.6	$d_{33} = 0.019 V^2$	$d_{50} = 0.031 V^2$
Basalt	2.9	$d_{33} = 0.016 V^2$	$d_{50} = 0.026 V^2$

8.3 Searcy (1967)

Searcy developed a rock-sizing chart for sizing riprap protection of waterway banks. His design chart can be represented by Equation 57. This equation would appear (relative to other publications) to be suitable only for high-density rock in low turbulent flow conditions.

$$d_{50} = 0.022 V^2 \quad (57)$$

8.4 J.P.Bohan (1970)

For sizing rock placed immediately downstream of culvert and pipe outlets, Bohan presents the following equations:

For low tailwater ($TW < D_o/2$): $d_{50} = 0.25 D_o F_o$ (58)

which can be converted to: $d_{50} = 0.080 D_o^{1/2} V_o$ (59)

For high tailwater ($TW > D_o/2$): $d_{50} = 0.25 D_o F_o - 0.15 D_o$ (60)

which can be converted to: $d_{50} = 0.080 D_o^{1/2} V_o - 0.15 D_o$ (61)

where:

TW = Tailwater depth relative to pipe invert [m]

D_o = internal diameter of outlet pipe [m]

F_o = Froude number of flow at the pipe exit [dimensionless]

8.5 Maynard (1978)

Maynard (1978) presented the following equations for sizing rock on channel beds and banks up to 3:1 (H:V) under flow conditions of deceleration flow.

Test result best fit: $d_{50} = 0.14 (SF)(y) F^{2.3}$ (62)

Recommended design: $d_{50} = 0.22 (SF)(y) F^3$ (63)

where:

SF = Recommended safety factor, SF = 1.15 (equivalent to a safety factor based on rock weight, $SF_w = 1.5$)

y = depth of flow within a trapezoidal channel [m]

F = Froude number based on flow depth (y) rather than hydraulic radius [dimensionless]

$$F = \frac{V}{\sqrt{g \cdot y}} \quad (64)$$

The above equations can be expanded to give:

Test result best fit: $d_{50} = \frac{0.14(SF)}{g^{1.15}} \left(\frac{V^{2.3}}{y^{0.15}} \right)$ (65)

Recommended design: $d_{50} = \frac{0.22(SF)}{g^{1.5}} \left(\frac{V^3}{y^{0.5}} \right)$ (66)

Now these equations can be compared to the traditional shear stress equations:

Shear stress equation:
$$d_{50} = \frac{SF.R.S_o}{F^*(s_r - 1)} \quad (67)$$

$$d_{50} = \frac{SF.n^2}{F^*(s_r - 1)} \left(\frac{V^2}{R^{1/3}} \right) \quad (68)$$

From the above it would appear that Maynard's original test results produced an equation that best aligns with the traditional shear stress equation. It is also noted that Maynard's analysis was performed on a trapezoidal channel where for average channel conditions may be defined by:

(typical channel conditions)
$$R = 0.6535(y) \quad (69)$$

thus,
$$d_{50} = \frac{1.152(SF)n^2}{F^*(s_r - 1)} \left(\frac{V^2}{y^{1/3}} \right) \quad (70)$$

An important outcome from this hydraulic study was the observation that for trapezoidal channels with side slope of 3:1 to 4:1, failure began to occur on the bed and banks at the same or similar flow conditions; however, for side slopes of 2:1 failure was always initiated on the banks. This indicates that for side slopes of 3:1 to 4:1 the combined effects of a decrease in shear stress on the banks plus interlocking forces between the rocks, is equally balanced by the increase in displacement forces resulting from gravity acting of the steep bank.

8.6 Bos, Replogle & Clemmens (1984)

Bos, Replogle, and Clemmens, (1984) present rock sizing equations suitable for rocks placed within the zone of highly turbulent water immediately **downstream** of the end sill of an energy dissipater (i.e. not within the main energy dissipation zone).

$$d_{40} = 0.038 V^{2.26} \quad (71)$$

If it is assumed that $d_{40}/d_{50} = 0.75$ (see Table 9), then:

$$d_{50} = 0.050 V^{2.26} \quad (72)$$

If it is assumed that Equation 72 is based on rock of a specific gravity (s_r) of 2.6, then:

$$d_{50} = \frac{(0.081).V^{2.26}}{(s_r - 1)} \quad (73)$$

8.7 Peterka (1984)

Peterka (1984) presented the following rock-sizing equation for riprap **downstream** of stilling basins:

$$d_{50} = 0.04 V^2 \quad (74)$$

8.8 Maynard (1988)

Maynard (1988) presented that following rock-sizing equation for rock with a gradation of $d_{85}/d_{15} = 1.8$ to 4.6 :

$$\frac{d_{30}}{y} = 0.30(\text{SF}) \left[\left(\frac{1}{(S_r - 1)} \right)^{0.5} \frac{V}{\sqrt{gy}} \right]^{2.5} \quad (75)$$

If it is assuming that $d_{30}/d_{50} = 0.55$ (Table 9):

$$d_{50} = \frac{0.031(\text{SF}) \left(\frac{V^{2.5}}{y^{0.25}} \right)}{(s_r - 1)^{1.25}} \quad (76)$$

8.9 Maynard, Ruff & Abt (1989)

Using dimensional analysis Maynard, Ruff & Abt (1989) suggested the following rock-sizing equation for rock placed on channel beds:

$$\frac{d}{y} = \text{constant} \left[\left(\frac{1}{(S_r - 1)} \right)^{0.5} \frac{V}{\sqrt{gy}} \right]^{2.5} \quad (77)$$

8.10 American Society of Civil Engineers (1992)

For the sizing of rocks for outlet protection immediately downstream of culvert and pipe outlets, ASCE (1992) presents the following equations:

$$d_{50} = \frac{0.066(Q)^{4/3}}{\text{TW}(D_o)} \quad (78)$$

For $\text{TW} = D_o/2$:

$$d_{50} = 0.0957 D_o^{2/3} V_o^{4/3} \quad (79)$$

where:

TW = tailwater depth relative to the outlet invert level [m]

D_o = internal diameter of the outlet pipe or culvert [m]

V_o = outlet velocity [m/s]

9.0 Rock sizing for the banks of low gradient (river) channels

9.1 Maynard, Ruff & Abt (1989)

Maynard, Ruff & Abt (1989) reported the following three key factors as being those factors that impact on rock stability within channel banks:

- Gravity component of rock weight acting down the bank slope.
- Influence of side slope on local velocity [and shear stress]. Steep side slopes often tend to move the maximum local velocity away from the toe of the slope (due to the effects of bank friction on bed velocity). Flatter side slopes allow the maximum velocity to occur over the toe of the slope. It is noted that flow velocity near the toe of the batter is important because bank rock located near the toe of the bank, can experience the combined effects of high shear stress plus the gravity component of its weight.
- Rock on steep banks experience interlocking (due to weight of up-slope rocks), which rock on the channel bed typically does not experience. On the channel bed the rocks behave more like individual units.

Test results showed that for side slopes of 1.5:1 to 3:1 (H:V) the coefficient associated with the rock sizing equation varied only slightly and thus they concluded the “limited influence of side slope angle on riprap size”. They recommend the same equation coefficient that is used for sizing rock of the channel bed, also be used for sizing rock on bank slopes equal to or less than 2:1, but a 25% increase in rock size should be applied for bank slopes of 1.5:1. Note this does **not** apply to rock placed on the face of steep gradient chutes.

Maynard, Ruff & Abt recommended a safety factor (applicable to rock size) of $SF = 1.2$. The resulting design equation is presented below as Equation 80. This equation has been developed for is rock with $d_{85}/d_{15} = 1.8$ to 4.6, assumed to be equivalent to, $d_{50}/d_{90} = 0.6$ to 0.23.

$$\frac{d_{30}}{y} = 0.30(SF) \left[\left(\frac{1}{(S_r - 1)} \right)^{0.5} \frac{V}{\sqrt{gy}} \right]^{2.5} \quad (80)$$

If it is assumed that $d_{30}/d_{50} = 0.55$, then:

$$d_{50} = \frac{0.031(SF)}{(s_r - 1)^{1.25}} \left(\frac{V^{2.5}}{y^{0.25}} \right) \quad (81)$$

It should be noted that Maynard, Ruff & Abt recommend the use of local average velocity rather than the average channel velocity—the latter being influenced by channel shape.

For channel bends, Maynard, Ruff & Abt referred to California Division of Highways (1970), which presented the following design equation:

$$\text{Design velocity on channel bend: } V_{\text{bend}} = (4/3) V_{\text{avg}} \quad (82)$$

where:

V_{bend} = the maximum depth-average velocity in the bend over the toe of the side slope

V_{avg} = the average channel velocity

However, for natural channels, Maynard (1988) recommends:

$$\text{Design velocity on channel bend: } V_{\text{bend}} = 1.5 V_{\text{avg}} \quad (83)$$

9.2 Standing Committee on Rivers and Catchments, Victoria (1991)

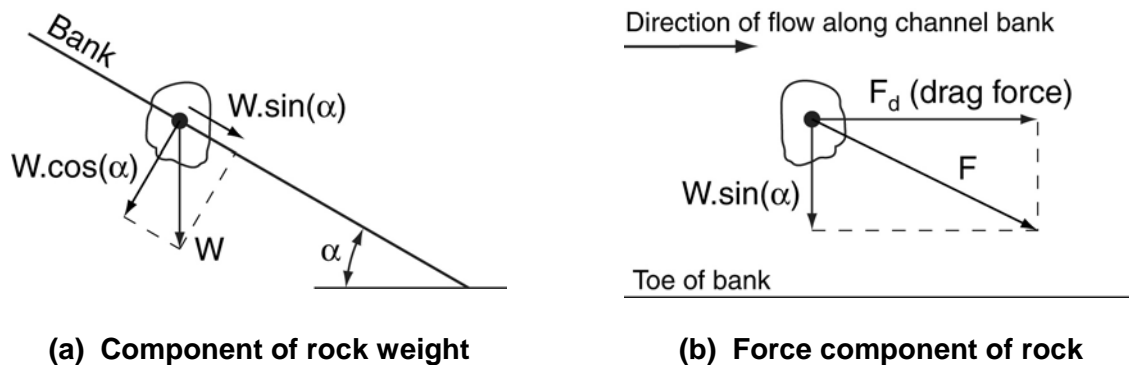


Figure 2 – Forces on rock located within the channel bank

The weight component down the face of the bank slope is given by Equation 84:

$$W.\sin(\alpha) \quad (84)$$

The resultant force (F) acting on the rock is given by Equation 85:

$$F^2 = W^2.\sin^2(\alpha) + (\text{drag force})^2 \quad (85)$$

The drag force is represented by Equation 86:

$$F_d = a.\tau_o \quad (86)$$

The resisting force (F_R) as suggested by Victoria Rivers (1991), Chow (1959) and Henderson (1966) is given by Equation 87:

$$F_R = W.\cos(\alpha).\tan(\Phi) \quad (87)$$

where:

- α = bank slope relative to the channel bed [degrees]
- F = resultant force on rock [N]
- F_d = drag force of rock [N]
- a = effective surface area [m^2]
- F_R = resisting force [N]
- Φ = angle of repose of rock [degrees]

If the design safety factor (SF_F) is adopted as the ratio of resisting force (F_R) divided by the resultant active force (F), then:

$$SF_F = F_R / F \quad (88)$$

$$SF_F = \frac{W.\cos(\alpha).\tan(\phi)}{\sqrt{(W^2.\sin^2(\alpha) + a^2.\tau_o^2)}} \quad (89)$$

For a flat bed (i.e. $\alpha = 0^\circ$) Shield's equation gives:

$$\tau_c = F^* \cdot \rho_w \cdot g \cdot d_{50} (s_r - 1) \quad (90)$$

If a safety factor of 1.0 is chosen (i.e. critical conditions), then:

$$SF_F = \frac{W \cdot \cos(0^\circ) \cdot \tan(\phi)}{\sqrt{(W^2 \cdot \sin^2(0^\circ) + a^2 \cdot \tau_c^2)}} = 1.0 \quad (91)$$

therefore;

$$a \cdot \tau_c = W \cdot \tan(\phi) \quad (92)$$

Equation 92 assumes that the effective shear stress surface area (a) can be directly related to the angle of repose, which is not necessarily the case.

Combining equations 90 and 92 gives:

$$a = \frac{W \cdot \tan(\phi)}{F^* \cdot \rho_w \cdot g \cdot d_{50} \cdot (s_r - 1)} \quad (93)$$

Also, Equations 21 and 22 provide the maximum shear stress on the bed and banks.

$$\text{Maximum shear stress on bed: } \tau_o = \rho_w \cdot g \cdot y \cdot \sin(\phi) \quad (94)$$

$$\text{Maximum shear stress on bank: } \tau_o = (0.75) \rho_w \cdot g \cdot y \cdot \sin(\phi) \quad (95)$$

Technical Note:

In the original Victoria Rivers (1991) derivation it was assumed that $\sin(\phi)$ in Equations 94 and 95 could be replaced by S_e however, this is only appropriate for low channel bed and bank slopes. Also, Vic Rivers indicate that shear stress on the bed is defined by, $\tau_o = (0.97) \rho_w \cdot g \cdot y \cdot S$.

The remainder of this section presents equations different from those presented within the original Victoria Rivers (1991) publication. Specifically, the term S_e has been replaced by $\sin(\phi)$, and the maximum shear stress expression for the channel bed, presented above as Equation 94 is adopted instead of the expression noted within the above technical note.

Thus Equation 89 becomes:

$$SF_F = \frac{W \cdot \cos(\alpha) \cdot \tan(\phi)}{\sqrt{(W^2 \cdot \sin^2(\alpha) + \left(\frac{\tau_o \cdot W \cdot \tan(\phi)}{F^* \cdot \rho_w \cdot g \cdot d_{50} (s_r - 1)} \right)^2)}} \quad (96)$$

which simplifies to:

$$SF_F = \frac{\cos(\alpha) \cdot \tan(\phi)}{\sqrt{(\sin^2(\alpha) + \left(\frac{\tau_o \cdot \tan(\phi)}{F^* \cdot \rho_w \cdot g \cdot d_{50} (s_r - 1)} \right)^2)}} \quad (97)$$

(a) Rock on channel bed

Given that the maximum shear stress on the channel bed is (Equation 94):

$$\tau_o = \rho_w \cdot g \cdot y \cdot \sin(\varphi) \quad (98)$$

Equation 97 becomes:

$$SF_F = \frac{\cos(\alpha) \cdot \tan(\phi)}{\sqrt{(\sin^2(\alpha) + \left(\frac{\rho_w \cdot g \cdot y \cdot \sin(\varphi) \cdot \tan(\phi)}{F^* \cdot \rho_w \cdot g \cdot d_{50} (s_r - 1)} \right)^2)}} \quad (99)$$

$$SF_F = \frac{\cos(\alpha) \cdot \tan(\phi)}{\sqrt{(\sin^2(\alpha) + \left(\frac{y \cdot \sin(\varphi) \cdot \tan(\phi)}{F^* \cdot d_{50} (s_r - 1)} \right)^2)}} \quad (100)$$

Also, for rock on the channel bed, $\alpha = 0^\circ$, thus:

$$SF_F = \frac{\cos(0^\circ) \cdot \tan(\phi)}{\sqrt{(\sin^2(0^\circ) + \left(\frac{y \cdot \sin(\varphi) \cdot \tan(\phi)}{F^* \cdot d_{50} (s_r - 1)} \right)^2)}} \quad (101)$$

$$SF_F = \frac{\tan(\phi)}{\sqrt{\left(\frac{y \cdot \sin(\varphi) \cdot \tan(\phi)}{F^* \cdot d_{50} \cdot (s_r - 1)} \right)^2}} \quad (102)$$

$$SF_F = \frac{F^* \cdot d_{50} (s_r - 1) \cdot \tan(\phi)}{y \cdot \sin(\varphi) \cdot \tan(\phi)} \quad (103)$$

$$SF_F = \frac{F^* \cdot d_{50} (s_r - 1)}{y \cdot \sin(\varphi)} \quad (104)$$

Rock on bed of channel/chute:
$$d_{50} = \frac{SF_F \cdot y \cdot \sin(\varphi)}{F^* (s_r - 1)} \quad (105)$$

For a low-gradient channel, $\sin(\varphi) \approx S_e$ and Equation 105 is effectively the same as the formula presented within Victoria Rivers publication.

It should be noted that Equation 105 is independent of the angle of repose of the rock (Φ).

(b) Rock on channel bank

Given that the maximum shear stress on the channel bank is (Equation 95):

$$\tau_o = (0.75) \cdot \rho_w \cdot g \cdot y \cdot \sin(\varphi) \quad (106)$$

Thus for rock on the **banks** of a channel Equation 97 becomes:

$$SF_F = \frac{\cos(\alpha) \cdot \tan(\phi)}{\sqrt{\sin^2(\alpha) + \left(\frac{(0.75) \rho_w \cdot g \cdot y \cdot \sin(\phi) \cdot \tan(\phi)}{F^* \cdot \rho_w \cdot g \cdot d_{50} (s_r - 1)} \right)^2}} \quad (107)$$

$$SF_F = \frac{\cos(\alpha) \cdot \tan(\phi)}{\sqrt{\sin^2(\alpha) + \left(\frac{(0.75) \cdot y \cdot \sin(\phi) \cdot \tan(\phi)}{F^* \cdot d_{50} (s_r - 1)} \right)^2}} \quad (108)$$

This can be rearranged to produce:

$$d_{50} = \frac{(0.75) \cdot y \cdot \sin(\phi)}{F^* \cdot \cos(\alpha) \cdot (s_r - 1) \sqrt{\left(\frac{1}{SF_F^2} - \frac{\tan^2(\alpha)}{\tan^2(\phi)} \right)}} \quad (109)$$

However, Equation 109 is based on the assumption that the resisting force (F_R) is given by Equation 87, which is not considered appropriate for very steep chutes. On steep chutes the effective maximum slope of the banks of the chute is a function of both the slope of the bank relative to the chute bed (α) plus the slope of the chute bed relative to the horizontal (θ).

It is noted that when the slope of the channel is significant, the gravitational force acting on a rock located on the channel bank is subject to a slope greater than the bank slope (i.e. the channel is now a complex three dimensional structure). The effective slope (β) of the rock weight component on a double slope formed by a bank slope (α) and a channel bed slope (θ) is given by Equation 110.

$$\tan^2(\beta) = \tan^2(\alpha) + \tan^2(\theta) \quad (110)$$

Therefore, the resisting force (F_R) is now given by Equation 111:

$$F_R = W \cdot \cos(\beta) \cdot \tan(\phi) \quad (111)$$

Thus Equation 109 now becomes:

$$\text{Rock on bank of chute: } d_{50} = \frac{(0.75) \cdot y \cdot \sin(\phi)}{F^* \cdot (s_r - 1) \sqrt{\left(\frac{\cos^2(\beta)}{SF_F^2} - \frac{\sin^2(\alpha)}{\tan^2(\phi)} \right)}} \quad (112)$$

If a low-gradient channel is assumed (i.e. $\sin(\phi) \approx S_e$, and $\cos(\beta) \approx \cos(\alpha)$), then Equation 112 becomes:

$$\text{(rock on channel bank)} \quad d_{50} = \frac{(0.75) \cdot y \cdot S_e}{F^* \cdot \cos(\alpha) \cdot (s_r - 1) \sqrt{\left(\frac{1}{SF_F^2} - \frac{\tan^2(\alpha)}{\tan^2(\phi)} \right)}} \quad (113)$$

For a low gradient channel Equation 113 is effectively the same as the formula originally presented by Victoria Rivers (1991).

10.0 Rock sizing for rock lined embankment experiencing overtopping flows

10.1 Isbash (1936)

As previously reported in Section 8.1, Isbash investigated the stability of rounded rock dumped into flowing rivers to for dams. This work focused on sizing individual stones (located on the downstream face of a dam) to resist displacement due to overtopping flow and percolation through the embankment.

The results of Isbash (1936) are presented in McLaughlin Water Engineers (1986) and reproduced below as Equation 114.

$$d_{50} = \frac{V^2}{2 \cdot g \cdot K^2 \cdot (s_r - 1)} \quad (114)$$

where:

K = equation constant based on flow conditions

Isbash recommended K = 0.86 for high turbulent river flow and K = 1.2 for low turbulent river.

10.2 Stephenson, D. (1979)

Stephenson (1979) performed a stability analysis for rocks placed on the downstream face of a rock embankment subject to overtopping flows. The results of Stephenson are reported in Abt and Johnson (1991).

$$d_{50} = \left[\frac{q(\tan\theta)^{7/6} n_p^{1/6}}{C g^{1/2} [(1 - n_p)(s_r - 1)(\cos\theta)(\tan\phi - \tan\theta)]^{5/3}} \right]^{2/3} \quad (115)$$

where:

q = threshold flow per unit width [m³/s/m]

n_p = porosity of rock embankment

C = coefficient (0.22 for gravel and pebbles, 0.27 for crushed stone)

s_r = relative density of rock

θ = slope of rock (embankment) face [degrees]

Φ = angle of repose of rock (angle of friction) [degrees]

Complete failure of the embankment is reported to occur when the unit discharge increased by 8% for crushed stone.

10.3 Abt, S.R. and Johnson, T.L. (1991)

Abt, S.R. and Johnson, T.L. (1991) investigated critical rock size (i.e. failure condition) for embankments subjected to overtopping flows. Bank slopes (S) of 1 to 20% were investigated.

The paper also presents a literature search that indicates Stephenson (1979) showed that due to the added interlocking of crushed rock, the required rock size (prior to threshold flow or incipient stone movement) is 87% of that required for natural rounded rock (river stone).

Abt and Johnson reported that “angular stone tends to interlock or wedge and subsequently resist sliding and rolling. In addition, fewer fines are required to fill the voids of crushed material

compared with a similarly graded rounded stone.”

The literature research showed that riprap designed to resist overtopping flows is a function of:

- relative rock size;
- irregularity of the rock (round or fractured quarry stone);
- the hydraulic gradient;
- the discharge (or unit discharge).

Abt and Johnson concluded that the critical rock size (for **angular crushed rock**), at which point failure begins to occur, is given by the following equation:

$$\text{(condition of total failure)} \quad d_{50,\text{failure}} = 0.5026 S_o^{0.43} q^{0.56} \quad (116)$$

where:

$d_{50,\text{critical}}$ = mean rock size (specific gravity = 2.65) at point of failure [m]

S_o = slope of rock protected embankment (range 1 to 20%) [m/m]

q = flow per unit width down the embankment [$\text{m}^3/\text{s}/\text{m}$]

It is important to note that the bed slope (S_o) is used in this equation, not the energy slope.

For incipient motion conditions the above equation becomes:

$$\text{(incipient rock movement)} \quad d_{50,\text{critical}} = 0.5950 S_o^{0.43} q^{0.56} \quad (117)$$

For design purposes a safety factor of 1.2 is recommended, thus the above equation becomes:

$$\text{(design purposes)} \quad d_{50} = 0.595 (\text{SF}) S_o^{0.43} q^{0.56} \quad (118)$$

$$\text{(for SF = 1.2)} \quad d_{50} = 0.714 S_o^{0.43} q^{0.56} \quad (119)$$

The above rock-sizing equations are based on a coefficient of uniformity, $C_u = d_{60}/d_{10} = 2.15$.

Abt and Johnson reported that for rounded rock, failure occurred at flow rates 32 to 45% lower than that for angular rock at a 10% embankment slope. This would result in a 17 to 23% increase in required rock size, i.e. angular rock would need to be 81 to 85% of rounded rock size, which is similar to the results of Stephenson (1979).

11.0 Rock sizing for rock-lined chutes

11.1 McLaughlin Water Engineers (Denver Guidelines, 1986)

The recommended design equation for rock on the bed of steep chutes as presented by McLaughlin Water Engineers (MWE) is reproduced below as Equation 120.

$$d_{50} = \frac{\tau}{F^* \rho_w \cdot g \cdot (s_r - 1) \cdot \cos(\theta) \cdot \left(\frac{1}{SF_F} - \frac{\tan(\theta)}{\tan(\phi)} \right)} \quad (120)$$

Equation 120 was further developed by MWE, but it is the author's opinion that their derivation contains errors as discussed below.

Technical note:

McLaughlin Water Engineers assumed that shear stress (τ) is represented by the following equation:

$$\tau = \rho_w \cdot g \cdot y \cdot S_e$$

But this assumes a low-gradient chute slope where $S_e = \tan(\phi) \approx \sin(\phi)$ which is **not** the case for a steep chute.

If the correct (in the author's opinion) expression for bed shear stress is inserted into Equation 120, then a rock-sizing equation in the form of Equation 121 is developed.

$$\text{Bed rock (non-uniform flow):} \quad d_{50} = \frac{y \cdot \sin(\phi)}{F^* \cdot \cos(\theta) \cdot (s_r - 1) \cdot \left(\frac{1}{SF_F} - \frac{\tan(\theta)}{\tan(\phi)} \right)} \quad (121)$$

$$\text{Noting that for non-uniform flow:} \quad \sin(\phi) \neq \sin(\theta) \quad (122)$$

McLaughlin Water Engineers also recommend the following parameters:

$$F^* = 0.091$$

$$SF_F = 1.5$$

$$\Phi = 42 \text{ degrees (assumed for large angular rock)}$$

If uniform flow conditions are assumed down the chute, such that the energy slope is parallel with the bed slope (i.e. $S = S_o = S_e$, and $\sin(\phi) = \sin(\theta)$), then Equation 121 becomes:

$$\text{Bed rock (uniform flow):} \quad d_{50} = \frac{y \cdot \tan(\theta)}{F^* \cdot (s_r - 1) \cdot \left(\frac{1}{SF_F} - \frac{\tan(\theta)}{\tan(\phi)} \right)} \quad (123)$$

McLaughlin Water Engineers recommended that rock size should be based on actual flow depth (y) above the rock rather than average flow depth (y_{ave}), or the hydraulic radius (R). Effectively the design is based on a 2-dimensional (2D) flow profile even though actual flow conditions may be highly 3-dimensional. The flow per unit width (q) is based on the hydraulic analysis of actual flow conditions at the crest (i.e. adopting the actual 3D crest profile).

12.0 Effects of the distribution of rock size on rock stability

The distribution of rock size assumed throughout this report for the conversion of reported rock-sizing equation based on d_x rock size to an equivalent d_{50} rock size is presented in Table 9.

Table 9 – Assumed distribution of rock size as applied in Table 6

Rock size ratio	Assumed distribution value
d_{100}/d_{50}	2.00
d_{90}/d_{50}	1.82
d_{75}/d_{50}	1.50
d_{65}/d_{50}	1.28
d_{40}/d_{50}	0.75
d_{33}/d_{50}	0.60
d_{10}/d_{50}	0.55

The distribution of rock size can also be described by the coefficient of uniformity, $C_u = d_{60}/d_{10}$. Poorly graded rock has a low value of C_u (say, $C_u < 2$), while well graded rock has a higher value of C_u (say, $C_u > 2.5$). Witter & Abt (1990) report that the recommended values of C_u specified by various published design guidelines generally falls in the range 1.1 to 2.70, but a typical value is around 2.1.

Witter & Abt investigated the effects of variations in the distribution of rock sizes on the stability of riprap. Their conclusions are summarised below:

- Poorly graded riprap has a high overall stability than well graded riprap.
- Following the displacement of individual surface rocks, well graded riprap tends to fill in the voids left by the displaced rocks through the movement of small particles washed from above, a process referred to as “healing”.
- Following the displacement of individual surface rocks, poorly graded riprap exhibits very little subsequent movement (rearranging) of rocks up until the point of mass failure.
- Failure of well graded riprap occurs more slowly than poorly graded riprap due to the “healing” action performed by the smaller rocks.
- Failure of poorly graded riprap begins with many surface rocks moving very suddenly.
- The aftermath of a keystone failure is a deep longitudinal scour in the embankment.
- A conservative approach to the sizing of rock should apply if the coefficient of uniformity (C_u) exceeds 3.0 [observation of Author following review of published test results].

In addition to the above, Witter & Abt report that Ahmed (1989) indicated that for rock placement on channel banks, poorly graded rock ($C_u \approx 1.1$) had a critical discharge 8% greater than well graded rock ($C_u \approx 2.2$). This would equate to allowing a 5% reduction in rock size for poorly graded rock compared to well graded rock when the rock is placed on the side slopes of a channel or chute.

13.0 Case studies and laboratory testing

13.1 Drop structures within the Denver, Colorado region

McLaughlin Water Engineers (1986) reported on field observations of various failed or stressed drop structures within the Urban Drainage and Flood Control District 69, Denver, Colorado, USA. A summary of the filed observations is presented in Table 10.

Table 10 – Case studies of sloping loose rock drop structures reported by McLaughlin Water Engineers (1986)

Site	Unit flow (m ³ /s/m)	Slope (H:1)	Drop height (m)	Mean rock size (mm)	Comments
Bear Canyon Creek Drop 5	1.04	4:1		450	Movement of few rocks
Cherry Creek	1.49	7:1		300 to 350	Expected failure point
Liberty Hill	1.96	4.5:1		< 600	Significant rock movement
Bear Canyon Creek Drop 9	2.08	4:1		380	Massive failure
Little Dry Creek, Westminster	2.34	5.35:1		450+	Some movement of small rocks
Lone Tree	2.74	11.5:1	0.6 to 0.9	300 to 380	Slight rock movement
Little Dry Creek, Georgetown	3.02	4.5:1		< 600	Some movement
Massey Draw	3.01	4:1	1.8	450	Rock clearly undersized for flow
Englewood Dam	4.38	8:1		450	Significant rock movement

13.2 Gay Street drop structure, Coopers Plains, Brisbane

Design conditions:

Design flow rate = 105m³/s

Design unit flow rate = 4.5m³/s/m

Chute slope = 15:1

Assumed distribution of rock size, $d_{50}/d_{90} = 0.5$

Design required mean rock size, $d_{50} = 750\text{mm}$ with $d_{10} = 600\text{mm}$

Trapezoidal approach channel and chute, with 1m wide, grouted rock low-flow channel passing through the elevated weir crest

The low-flow channel does not continue down the face of the chute

Crest width (at invert) = 23m

Design side slope of 2.5:1 one side and 3.5:1 other side (at crest)

Drop height from crest to downstream channel invert = 2.54m

Post storm survey of structure:

Constructed weir crest profile was surveyed (Figure 3)

Mean rock size estimated at 600mm +/- 100mm

Movement in the 600mm rock appears to have been a direct response to the loss of the smaller rocks

Rocks moved well down the channel by flow were up to 400mm, but larger rock were displaced as a result of total failure of the structure

Estimate the critical rock size to be 500mm (i.e. average of 400mm and 600mm)

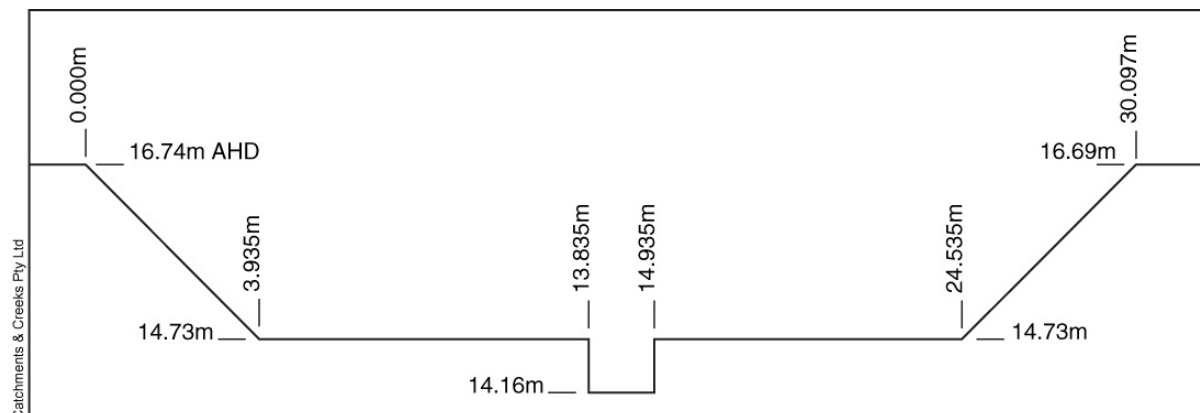


Figure 3 – Cross section (xs573) of chute crest, Gay Street drop structure

Destructive flow event, 9 March 2001:

Estimated peak discharge = 105m³/s (approx 1 in 30 year event), but could be in the range 92 to 120m³/s

Constructed weir crest profile was surveyed (Figure 3) and used in the hydraulic analysis

Critical depth at crest (xs573) at 105m³/s = 15.99m AHD, flow area = 31.33m², top width = 27.76m, average velocity = 3.35m/s

Table 11 – HecRas analysis of 9 March 2001 storm

Cross-section	Chainage	Water elevation (AHD)
XS 653	653m	16.77m
XS 573 (crest)	573m	15.99m
XS 453	453m	14.7m

Storm conditions at critical cross sectional flow conditions (XS563):

Energy slope = 0.06676

Flow depth = 1.293m

Calculated unit flow rate = 5.0m³/s/m

Estimated Manning's roughness = 0.081

Calculated velocity = 3.45m/s

Table 12 – Case study of Gay Street, Coopers Plains, Brisbane

Site	Unit flow (m ³ /s/m)	Slope (H:1)	Drop height (m)	Mean rock size (mm)	Comments
Gay Street, Brisbane	5.0	15:1	2.5	600-400	Assumed critical rock size = 500mm



Photo 1 – Failed drop structure seven months after 1 in 30yr storm



Photo 2 – Failed structure with concrete weir visible and undamaged



Photo 3 – Repaired structure (grouted rock)



Photo 4 – Low-flow channel through weir crest (note the component of the drop was not damaged due to original concrete grouting)

13.3 Physical model studies reported by Abt and Johnson (1991)

Abt and Johnson (1991) reported on flume studies of rock embankments experiencing overtopping flows.

Further to the results reported by Abt and Johnson, the following analysis was performed by the author. This analysis has been based on the following assumptions and calculations:

- Distribution of rock size (d_{50}/d_{90}) was estimated from the reported (d_{84}/d_{16}) value.
- Manning's roughness was determined through the application of Equation 11.
- Flow depth was calculated to achieve the reported unit flow rate for the assumed Manning's roughness, effective hydraulic radius assumed equal to the flow depth, and energy slope equal to the embankment slope.

Table 13 – Summary of laboratory test results by Abt and Johnson (1991)

Rock shape	d50	Slope	flow rate (failure)	Specific weight	d50/d90	Manning	Depth
	(m)	(-)	(m ³ /s/m)	–	–	–	m
	Reported values				Calculated values		
Angular	0.104	0.20	0.168	2.65	0.249	0.169	0.193
Angular	0.130	0.20	0.331	2.65	0.288	0.139	0.257
Angular	0.157	0.20	0.412	2.65	0.289	0.146	0.303
Angular	0.056	0.20	0.046	2.72	0.258	0.162	0.086
Rounded	0.102	0.20	0.088	2.65	0.198	0.254	0.166
Rounded	0.102	0.20	0.088	2.65	0.198	0.254	0.166
Angular	0.051	0.10	0.079	2.72	0.215	0.145	0.138
Angular	0.051	0.10	0.093	2.72	0.215	0.139	0.147
Angular	0.051	0.10	0.103	2.72	0.215	0.135	0.155
Rounded	0.102	0.10	0.185	2.50	0.240	0.154	0.238
Rounded	0.102	0.10	0.194	2.50	0.240	0.153	0.243
Angular	0.102	0.10	0.326	2.65	0.198	0.163	0.347
Angular	0.102	0.10	0.352	2.65	0.198	0.160	0.359
Rounded	0.051	0.10	0.064	2.72	0.094	0.367	0.210
Angular	0.102	0.10	0.383	2.65	0.198	0.156	0.373
Angular	0.056	0.20	0.421	2.72	0.258	0.093	0.236
Angular	0.026	0.20	0.103	2.72	0.301	0.073	0.087
Angular	0.026	0.01	0.139	2.72	0.301	0.049	0.202
Angular	0.026	0.10	0.033	2.72	0.301	0.089	0.061
Angular	0.026	0.10	0.032	2.72	0.301	0.090	0.060
Angular	0.026	0.10	0.029	2.72	0.301	0.092	0.057
Angular	0.026	0.10	0.039	2.72	0.301	0.085	0.066
Angular	0.056	0.10	0.104	2.72	0.258	0.120	0.146
Angular	0.056	0.10	0.116	2.72	0.258	0.118	0.154
Angular	0.056	0.10	0.116	2.72	0.258	0.118	0.154
Angular	0.056	0.08	0.168	2.72	0.258	0.104	0.191

Table 14 – Average and range of test results and parameters for the laboratory testing carried out by Abt and Johnson (1991)

	Specific gravity	Mean rock size	Slope	Unit flow at failure	Assumed depth	y/d ₅₀	Critical Shields ^[1]
	–	(mm)	(m/m)	(m ³ /s/m)	(m)	–	–
Minimum	2.50	26	0.01	0.029	0.057	1.54	0.040
Average	2.68	69	0.127	0.162	0.184	2.88	0.194
Maximum	2.72	157	0.2	0.421	0.373	7.77 ^[2]	0.311

[1] The equivalent Shield's parameter has been determined using the equation presented by McLaughlin Water Engineers (Section 11.1) based on a safety factor, SF = 1.0 (i.e. critical conditions).

[2] The second highest value was, y/d₅₀ = 4.21.

13.4 Physical model studies reported by Peirson et al. (2008)

Peirson et al. (2008) conducted physical model studies on both randomly dumped and placed rock layered directly on the bed of a tilting flume subject to overtopping flows. The rock and placement properties are summaries in Table 15.

Table 15 – Summary of rock and placement properties

Description	Sandstone		Sandstone		Basalt	
Specific gravity (s_r)	2.29		2.37		2.64	
Mean diameter (d_{50}) [mm]	76		109		94	
Coefficient of uniformity (C_u)	1.18		1.14		1.21	
Rock placement properties:						
Configuration	Random	Placed	Random	Placed	Random	Placed
Packing number [rocks/m ² /layer]	145	187	71	90	93	117
Porosity	0.47	0.39	0.44	0.38	0.47	0.37
Depth of two layers of rock [mm]	121	141	174	204	151	184
Unit density of two layers [kg/m ³]	1230	1410	1320	1450	1405	1717
Angle of repose ^[1] [degrees]	44	49	50	49	48	53

[1] Determined from an average of three observations of the angle of tilt required to induce rock displacement on a test ramp with an underlayer of 40mm basalt fixed to the ramp floor.

Failure was judged as the condition where the rock armour exposed the filter material. The total discharge being the sum of the flow carried within the rock blanket (thus subject to the thickness and porosity of the rock layer) and the flow over the rock blanket.

A summary of the test results is presented in Table 16.

Table 16 – Summary of test results

Bed slope (m/m)	Sandstone (76mm)			Sandstone (109mm)			Basalt (94mm)		
	0.2	0.3	0.4	0.2	0.3	0.4	0.2	0.3	0.4
Random placement									
$q_{failure}$ [m ² /s]	0.118	0.084	0.054	0.233	0.156	0.156	0.194	0.183	0.161
Placed rock									
$q_{failure}$ [m ² /s]	0.156	0.147	0.124	0.308	0.245	0.201	N/A	0.271	0.256
$q_{(placed)}/q_{(random)}$	1.32	1.75	2.29	1.32	1.57	1.29	N/A	1.48	1.59

Other outcomes from this laboratory testing include:

- A typical 30% increase in the design flow for placed rock compared to randomly placed rock, but an approximate 35% increase in the density of rock used to form two layers of placed rock.
- Average thickness of two layers of rock were $1.60(d_{50})$ for randomly placed rock and $1.86(d_{50})$ for placed rock.
- Flow must not exceed 50% of the failure discharge if substantial rearrangement of the armour rock is to be avoided.

14.0 Discussion

The following section is presented for discussion purposes only. It does not necessarily represent the final recommendations, which are provided in Section 15.

14.1 Use of average or local velocity

Rock displacement occurs as a result of local forces, local shear stresses, and local velocities. Therefore, it would not appear reasonable to base rock size on flow velocities averaged over the full cross section, or to relate it to the hydraulic radius of the cross section, which is a product of the 2-dimensional aspects of a channel section.

It would appear reasonable to base rock size on an assumed 1-dimensional flow pattern that considers unit flow rate (q) instead of total flow (Q), local flow depth (y) instead of hydraulic radius (R), and local depth-average velocity instead of average cross-sectional velocity.

14.2 Factor of safety

It would appear from the literature that one of the most common causes of the failure of loose rock chutes relates to the failure to construct the chute from rock of the specified size. It would appear unrealistic for the designer to assume that rock can always be supplied to the exact sizes specified. Therefore, it would appear reasonable that a safety factor (relative to rock size) greater than 1.2 should be specified if there is expected to be poor construction supervision. As a minimum, a safety factor of 1.5 should be applied unless the designer knows with certainty that a specific rock size and size distribution can be supplied.

14.3 Effects of air entrainment

Peirson et al. (2008) carried out model studies on steep rock-lined spillways operating in partial whitewater conditions where air entrainment within the overtopping flow becomes an important component of the destabilising forces acting on the rocks.

Under these conditions the effective flow density decreases below that of ambient water due to air entrainment. As a result, the effective flow depth increases and the forces exerted on the rocks by the fluid decreases. In addition, rock stability is increased due to the reduced effect of buoyancy on submerged rock.

The results of Peirson et al. appear to demonstrate that traditional shear stress based rock sizing equations significantly over estimate rock size for steep gradient chutes and spillways operating under whitewater conditions.

14.4 Effects of interflow

Peirson et al. (2008) discusses the importance of the interflow component relative to the total overtopping flow. Interflow is that portion of the total flow that moves within the rock armour. Thus the total overtopping flow is defined as:

$$\text{Total flow} = \text{overflow} + \text{interflow}$$

This interflow region is typically defined from the upper surface of the underlying filter layer up to an elevation $d_{50}/3$ below the upper surface of the armour rock face.

Interflow decreases with decreasing porosity and depth of the armour layer. Porosity decreases with increasing coefficient of uniformity ($C_u = d_{60}/d_{10}$) as the rock moves from poorly graded to well graded rock. Porosity also decreases with increasing sedimentation of the air voids.

14.5 Aging of structures

The failure of loose rock structures is contributed to by the upward pressure of water passing under individual rocks, and by the water pressure acting on the exposed face of the rock. Observations by the author indicate that the majority of failures of rock chutes have occurred to chutes that have only recently been constructed. In such chutes the voids are likely to remain primarily open to air and water movement. Once these voids become blocked with sediment and the sediment is stabilised with vegetation (ground covers), the stability of the structure appears to increase.

Thus for structures designed for a long service life based on a design storm frequency significantly greater than say 1 in 10 years, it would be expected that the structure is unlikely to experience a near-design flow before the structure has been further stabilised by sediment and vegetation. However, for short-term structures, such as rock-lined spillways associated with construction site sediment basins, the risk of a near-design flow occurring while the structure is still acting as an open void, loose rock structure is high, and thus a greater factor of safety should be adopted.

14.6 Manning's roughness of riprap

A significant number of the rock-sizing theories have been based on an assumed Manning's roughness determined from Strickler's equation. Unfortunately, this equation is only considered appropriate for deepwater flow conditions. Thus care must be taken when interpreting any equations and test results based on the use of the Strickler's equation if shallow water conditions exist. In some cases, re-analysis is required based on a more appropriate Manning's roughness value.

An alternative method for determining the Manning's roughness of rock in deep and shallow water is proposed within this paper.

14.7 Assessment of complex bank slopes

In cases where the channel or chute bed slope is steep (say $S_o > 10\%$), the effective slope of the banks relative to the horizontal can be significantly greater than the bank slope relative to the bed horizontal (i.e. the bank slope shown within a given channel/chute cross-section). The gradient of this complex slope (β) is given by Equation 124.

$$\tan^2(\beta) = \tan^2(\alpha) + \tan^2(\theta) \quad (124)$$

where:

- β = bank slope relative to the horizontal [degrees]
- α = bank slope relative to the channel bed [degrees]
- θ = slope of channel bed [degrees]

14.8 Application of rock-sizing equations based on shear stress for rock on the bed of low-gradient channels

As noted in Section 7.6, Maynard, Ruff & Abt (1989) reported that investigations by Meyer-Peter & Muller 1948, Blench 1966, Neill 1967 and Bogardi 1978, have shown *“that the Shield's coefficient is not constant as used in many riprap design procedures, but varies directly with relative roughness (particle size/depth)”*. The reported values of the critical Shield's parameter presented in Tables 6 appear to support this conclusion.

A discussion on the applicability of shear stress to rock stability, and possible reasons why the observed critical shear stress varies with depth, are presented below.

In laminar flow conditions, shear stress consists of a force transfer between individual moving layers as a result of water viscosity and the velocity gradient. There is very little flow transfer between the layers of water because agitation of the fluid particles exists only at a molecular level. In riprap design, laminar flow conditions would only exist deep within the riprap layer as inter-void flow.

In turbulent flow, there is both energy and mass transfer between the various 'layers' of water. In reality, however, it is inappropriate to refer to 'layers' of water existing within turbulent flow because of the highly 3-dimensional flow paths, and the rapid transfer of water between different elevations within the water column.

The concept of shear stress within turbulent flow may be treated as a simple energy or force transfer between water at different elevations so long as the depth of flow is significantly greater than the size of the roughness units, say, $y/d_{50} > 10$. However, in circumstances where the depth of flow approaches the size of the roughness units, then a highly irregular water surface develops and the flow takes on the characteristics commonly referred to as whitewater. In such conditions, the surface rocks are subjected to highly variable flow conditions, with macro-size jets randomly impacting upon the leading surface of the exposed rocks.

In whitewater conditions, the exposed rocks are subjected more to the displacing forces of tumbling jets of water, than what is traditionally referred to as shear stress. An effective shear stress parameter can still be calculated from the test results, but the values are much higher than those experienced under deepwater conditions.

Also, under whitewater conditions, the effects of air entrainment reduce the effective shear stress as discussed in Section 14.3.

Under **non-uniform flow conditions** on a low-gradient channel, Equation 48 is considered the most appropriate rock-sizing equation based on shear stress:

$$\text{(non-uniform, low gradient flow)} \quad d_{50} = \frac{SF \cdot R \cdot S_e}{F^* (s_r - 1)} \quad (125)$$

where:

d_{50} = the 'design' mean rock size based on application of a suitable safety factor (SF) to the critical rock size [m]

The critical shear stress decreases with increasing slope due to the effects of the increasing instability of a sloping rock face as the slope approaches the natural angle of repose of dumped rock. However, if the theoretical effects of embankment slope are considered separate to the shear stress, then the effective critical shear stress actually increases with increasing embankment slope due to the increasing stabilising effects of interlocking between individual rocks with increasing slope, and the effects of air entrainment.

If the theoretical effects of embankment slope are included into the shear stress equation, then the most widely adopted rock-sizing equation takes the form of Equation 126.

$$\text{(non-uniform, steep gradient flow)} \quad d_{50} = \frac{R \cdot S_e}{F^* (s_r - 1) \left(\frac{1}{SF} - \frac{\tan(\theta)}{\tan(\phi)} \right)} \quad (126)$$

In Equation 126, the Shield's parameter (F^*) is not considered a constant, but increases with increasing embankment slope.

14.9 Application of velocity-based rock-sizing equations for rock on the bed of low-gradient channels

The wide variation in reported velocity-based rock sizing equations is most likely the result of the wide variation in flow conditions investigated by the various authors, and the difficulty of developing a single equation that can address a wide range of possible flow conditions. Most of the equations take the form of either Equations 127 or 128 depending on bed slope.

$$\text{Low gradient flow:} \quad d_{50} = \frac{\text{SF} \cdot A \cdot V^B}{(s_r - 1)} \quad (127)$$

$$\text{Steep gradient flow:} \quad d_{50} = \frac{A \cdot V^B}{(s_r - 1) \left(\frac{1}{\text{SF}} - \frac{\text{Tan}(\theta)}{\text{Tan}(\phi)} \right)} \quad (128)$$

Typical values of the constants 'SF.A' and 'B' reported in the literature for use in Equations 127 and 128 are presented in Table 17.

Table 17 – Typical values of equation constants (SF.A) and (B)

Flow conditions	SF.A	B
Deepwater, low turbulent (straight river reach)	0.035	2
Deepwater, highly turbulent flow (low gradient rock chutes)	0.069	2
Immediately downstream of hydraulic jumps and energy dissipaters	0.081	2.26

Maynard (1978) presented the following equation that included flow depth. This equation has a number of similarities with the shear stress equations.

$$d_{50} = \frac{0.14(\text{SF})}{g^{1.15}} \left(\frac{V^{2.3}}{y^{0.15}} \right) \quad (129)$$

If it can be assumed that Maynard's results relate to rock of a specific gravity of 2.6, then the above equation can be converted to:

$$d_{50} = \frac{0.018(\text{SF})}{(s_r - 1)^{1.25}} \left(\frac{V^{2.3}}{y^{0.15}} \right) \quad (130)$$

Using dimensional analysis Maynard, Ruff & Abt suggest the following rock-sizing equation for rock on channel beds:

$$\frac{d}{y} = \text{constant} \left[\left(\frac{1}{(s_r - 1)} \right)^{0.5} \frac{V}{\sqrt{gy}} \right]^{2.5} \quad (131)$$

which can be re-arranged as:

$$d_{50} = \frac{\text{constant} \cdot (\text{SF})}{(s_r - 1)^{1.25}} \left(\frac{V^{2.5}}{y^{0.25}} \right) \quad (132)$$

Maynard (1988) finally presented the above equation as:

$$d_{50} = \frac{0.031(SF)}{(s_r - 1)^{1.25}} \left(\frac{V^{2.5}}{y^{0.25}} \right) \quad (133)$$

The above equations indicate that a flow depth term (y) may be required in these velocity-based equations in order to allow the equations to be applicable over a wide range of flow conditions. Thus the standard velocity-based rock sizing equations would likely take the form of Equations 134 and 135.

Low gradient flow:
$$d_{50} = \frac{SF \cdot A \cdot V^B}{y^C (s_r - 1)} \quad (134)$$

Steep gradient flow:
$$d_{50} = \frac{A \cdot V^B}{y^C (s_r - 1) \left(\frac{1}{SF} - \frac{\tan(\theta)}{\tan(\phi)} \right)} \quad (135)$$

The work of Peirson et al. (2008) suggests that there may also be the need to incorporate an additional bed slope term into these equations to address the effects of air entrainment when they are applied to steep slopes.

14.10 Application of rock sizing equation for rock on channel banks

Maynard, Ruff & Abt (1989) recommended that the same equation coefficient that is used for sizing rock of the channel bed, should also be used for sizing rock on bank slopes equal to or less than 2:1, but a 25% increase in rock size should be applied for bank slopes of 1.5:1. Note, this only applies to rock on the banks of channels and chutes, **not** to the rock placed on the face of steep gradient chutes.

Maynard, Ruff & Abt (1989)
$$d_{50} = \frac{0.031(SF)}{(s_r - 1)^{1.25}} \left(\frac{V^{2.5}}{y^{0.25}} \right) \quad (136)$$

It should be noted that Maynard, Ruff & Abt recommend the use of local average velocity rather than the average channel velocity.

For channel bends, California Division of Highways (1970), which presented the following:

$$V_{\text{bend}} = (4/3) V_{\text{avg}} \quad (137)$$

However, for natural channels, Maynard (1988) recommends:

$$V_{\text{bend}} = 1.5 V_{\text{avg}} \quad (138)$$

14.11 Application of rock sizing equation for bed rock on steep gradient channels and chutes

The results of Isbash (1936) are presented in McLaughlin Water Engineers (1986) and reproduced below as Equation 139.

$$d_{50} = \frac{V^2}{2 \cdot g \cdot K^2 \cdot (s_r - 1)} \quad (139)$$

Stephenson (1979) suggested the following:

$$d_{50} = \left[\frac{q(\tan \theta)^{7/6} \eta_p^{1/6}}{Cg^{1/2} [(1 - \eta_p)(s_r - 1)(\cos \theta)(\tan \phi - \tan \theta)]^{5/3}} \right]^{2/3} \quad (140)$$

Stephenson showed that due to the added interlocking of crushed rock, the required rock size (prior to threshold flow or incipient stone movement) is 87% of that required for natural rounded rock (river stone).

Abt and Johnson (1991) reported that “angular stone tends to interlock or wedge and subsequently resist sliding and rolling. In addition, fewer fines are required to fill the voids of crushed material compared with a similarly graded rounded stone.” Abt and Johnson presented the following design equation for rock on the downstream face of a rock embankment subjected to overtopping flows:

$$d_{50} = 0.595 (SF) S_o^{0.43} q^{0.56} \quad (141)$$

Abt and Johnson also reported that for rounded rock, failure occurred at flow rates 32 to 45% lower than that for angular rock at a 10% embankment slope. This would result in approximately a 40% increase in required rock size for rounded rock.

McLaughlin Water Engineers (1986) presented the following equation for rock chutes. This equation has the benefit of including a safety factor directly linked to the angle of the bed slope relative to the natural angle of repose of the dumped rock.

$$d_{50} = \frac{y \cdot \sin(\phi)}{F \cdot \cos(\theta) \cdot (s_r - 1) \cdot \left(\frac{1}{SF_F} - \frac{\tan(\theta)}{\tan(\phi)} \right)} \quad (142)$$

14.12 Determination of the best equation formats for assessment against the field and laboratory data

Various sets of field and laboratory test data for rock chutes are presented in Section 13 of this paper. The various rock-sizing equations presented above were tested on this data to determine the best-fit equation.

Reviewing Sections 7 through to 10 reveals that the equations presented for sizing rock in deepwater, low gradient channels generally take the form of the following three equations:

$$d_{50} = \frac{SF \cdot R \cdot S_e}{F \cdot (s_r - 1)} \quad (143)$$

$$d_{50} = \frac{SF \cdot V^2}{2 \cdot g \cdot K^2 \cdot (s_r - 1)} \quad (144)$$

$$d_{50} = \frac{0.031(SF)}{(s_r - 1)^{1.25}} \left(\frac{V^{2.5}}{y^{0.25}} \right) \quad (145)$$

In order for the above deep-water, low-gradient equations to blend seamlessly with the shallow-water, steep-gradient equation presented by Abt and Johnson, it would be desirable for the deep-water equations to be converted to a form similar to Equation 146:

$$d_{50, \text{critical}} = \frac{K \cdot K_1 \cdot K_2 \cdot S_e^A \cdot q^B}{(s_r - 1)} \quad (146)$$

where:

$d_{50, \text{cri}}$ = critical (SF = 1) mean rock size [m]

K = equation constant

K_1 = correction factor for rock shape

= 1.0 for angular (fractured) rock, 1.36 for rounded rock (i.e. smooth, spherical rock)

- K_2 = correction factor for rock grading
 = 0.95 for poorly graded rock ($C_u = d_{60}/d_{10} < 1.5$), 1.05 for well graded rock ($C_u > 2.5$),
 otherwise $K_2 = 1.0$ ($1.5 < C_u < 2.5$)
 S_e = slope of energy line at given cross-section [m/m]
 q = unit flow rate based on local flow depth at a given location [$m^3/s/m$]
 s_r = specific gravity of rock

If we consider 2D flow conditions where the hydraulic radius (R) is equal to flow depth (y), then the following applies:

$$V = \frac{R^{2/3} S_e^{1/2}}{n} \quad (147)$$

and

$$q = \frac{R^{5/3} S_e^{1/2}}{n} \quad (148)$$

Therefore Equations 143 to 145 can be transformed as follows:

$$d_{50} = \frac{\text{constant} \cdot R \cdot S_e}{(s_r - 1)} \quad (149)$$

$$d_{50} = \frac{\text{constant} \cdot R^{4/3} \cdot S_e}{(s_r - 1)} \quad (150)$$

$$d_{50} = \frac{\text{constant} \cdot R^{17/12} \cdot S_e^{5/4}}{(s_r - 1)^{1.25}} \quad (151)$$

Thus Equations 149 to 151 can be presented in the form:

$$d_{50} = \frac{\text{constant} \cdot R^C \cdot S_e^D}{(s_r - 1)^E} \quad (152)$$

where:

$$1.0 < C < 1.42$$

$$1.0 < D < 1.25$$

$$1.0 < E < 1.25$$

and:

$$C = \frac{5B}{3} \quad (153)$$

$$D = A + 0.5B \quad (154)$$

Therefore, to obtain an equation that works for shallow water, deep water, and low and steep gradient bed conditions, it would 'appear' desirable for the power constants A and B to fall within the range:

$$0.575 < A < 0.95$$

$$0.6 < B < 0.85$$

14.13 Determination of the best-fit equation based on the field and laboratory data

A best-fit analysis of the field and laboratory data produced the following equation:

$$\text{(uniform flow)} \quad d_{50,\text{critical}} = \frac{0.787 \cdot K_1 \cdot K_2 \cdot S_o^{0.35} \cdot q^{0.63}}{(s_r - 1)} \quad (155)$$

However, the data set contains some field observations from Denver where the actual rock size was noted as being clearly undersized, or the discharge was significantly greater than the design storm. The three sites in question are listed in Table 18.

Table 18 – Questionable field data

Site	Comments recorded in original report
Bear Canyon Creek Drop 5	<ul style="list-style-type: none"> • Movement of few rocks • Rocks clearly smaller than that specified.
Bear Canyon Creek Drop 9	<ul style="list-style-type: none"> • Massive failure
Massey Draw	<ul style="list-style-type: none"> • Rock clearly undersized for flow • Rocks clearly smaller than that specified. • Rocks are poorly graded with subgrade open to flow

The above three data points were removed from the data set and the best-fit analysis was repeated. Based on this reduced data set the following revised best-fit equation was developed.

$$\text{(uniform flow)} \quad d_{50,\text{critical}} = \frac{0.81 \cdot K_1 \cdot K_2 \cdot S_o^{0.36} \cdot q^{0.63}}{(s_r - 1)} \quad (156)$$

Plots of the observed d_{50} rock size compared to that calculated by the relevant best-fit equations are presented as Figure 4 and 5.

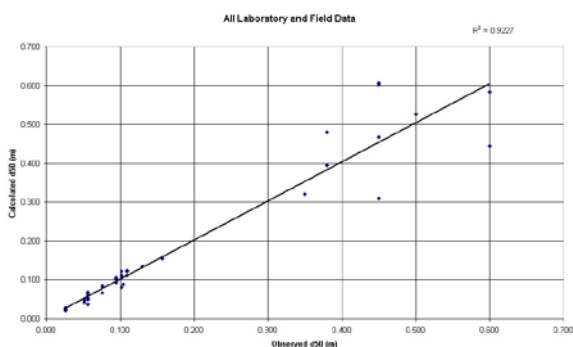


Figure 4 – Observed rock size vs calculated rock size using full data set

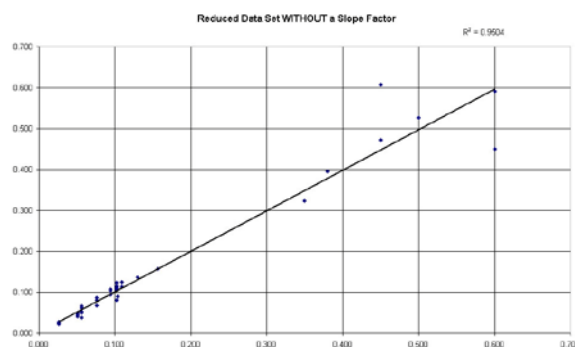


Figure 5 – Observed rock size vs calculated rock size using reduced data set

As can be seen from Equation 155, the power constant applied to the unit flow rate (q) is 0.63, which is inside the expected range of 0.6 to 0.85 (see Section 14.12). However, the power constant applied to the bed slope (S_o) of 0.35 is below the expected range of 0.575 to 0.95.

Using Equation 156 developed from the reduced data set, the power constant applied to the bed slope (S_o) of 0.36 is also below the expected value of 0.575 to 0.95; however, the power constant applied to the unit flow rate (q) remains at 0.63, which is inside the expected range of 0.6 to 0.85.

The above results imply that the bed slope (S_o) may not be as influential in determining rock size as previously thought. It is noted that both energy slope and bed slope are considered here because the equations analysed assume uniform flow conditions where energy slope and bed slope are equal. However, logic would suggest that it is in fact the reduced influence of bed slope. Possible reasons for this reduced influence of bed slope include:

- The effects of interlocking between individual rocks resulting from the weight of up-slope rocks impinge upon the lower rocks thus increasing their stability.
- The effects of air entrainment on steep slopes.
- Those equations based on shear stress assume that under uniform flow conditions (i.e. the flow reached a constant velocity where energy slope equals bed slope) the riprap exerts a force on the water equal to the weight of the water. However, this assumption becomes increasingly questionable with increasing bed slope.

Equations 155 and 156, however, do not necessarily take the preferred equation format because neither equation incorporates the angle of repose of the rock, which is considered significant at steep bed slopes. Instead, the effects of the angle of repose are directly incorporated into the bed slope power factor through laboratory testing of steep rock-lined chutes and spillways.

Also, neither equation is appropriate for non-uniform flow when bed slope (S_o) and energy slope (S_e) are not equal.

14.14 Determination of the best-fit equation based on the field and laboratory data supplemented with additional, deep-water data points

It was noted that the majority of the field and laboratory data is based on shallow water conditions relative to the size of the rocks. If the ultimate equations developed from this project are going to be flexible enough to be used in both deep and shallow water conditions, then it was considered necessary to supplement the existing data set with some information (data points) appropriate for deepwater conditions.

To determine these additional data points, the work of the following authors was considered:

- McLaughlin Water Engineers (Denver Guidelines, 1986)
- Maynard (1978)
- Maynard, Ruff & Abt (1989)
- Abt, S.R. and Johnson, T.L. (1991)

After comparing the rock sizes determined from the various equations presented by the above authors it was decided that a set of a low bed slope, deepwater data points could be obtained from an average of the standard Shield's equation with a Shield's parameter of 0.07, and the recommended design equation presented by Maynard (1978).

Table 19 provides a summary of key parameters associated with the original field and laboratory data points. Tables 20 and 21 provide a summary of key parameters associated with the additional low-gradient, deepwater data points.

Table 19 – Adopted data parameters used in testing the equations

Location	d90 (m)	Manning's roughness	Local velocity m/s)	Unit discharge	K1	K2
Bear No.5 ^[1]	0.910	0.121	2.38	1.04	1.00	1.00
Cherry Ck	0.796	0.098	2.55	1.49	1.00	1.00
Liberty Hill	1.250	0.121	2.92	1.96	1.00	1.00
Bear No.9 ^[1]	1.160	0.125	2.73	2.08	1.00	1.00
Little Dry Ck	1.150	0.112	2.94	2.34	1.00	1.00
Lone Tree	0.930	0.087	2.95	2.74	1.00	1.00
Georgetown	1.520	0.125	3.22	3.02	1.00	1.00
Massey D. ^[1]	1.450	0.130	3.04	3.01	1.00	1.00
Englewood	1.340	0.102	3.42	4.38	1.00	1.00
Gay St	1.300	0.086	3.56	5.00	1.00	1.00
Sandstone	0.095	0.050	1.59	0.12	1.00	0.95
Sandstone	0.095	0.056	1.45	0.08	1.00	0.95
Sandstone	0.095	0.065	1.22	0.05	1.00	0.95
Sandstone	0.136	0.051	2.05	0.23	1.00	0.95
Sandstone	0.136	0.059	1.81	0.16	1.00	0.95
Sandstone	0.136	0.061	1.94	0.16	1.00	0.95
Basalt	0.118	0.050	1.94	0.19	1.00	0.95
Basalt	0.118	0.053	2.07	0.18	1.00	0.95
Basalt	0.118	0.056	2.06	0.16	1.00	0.95
Angular	0.462	0.117	1.28	0.17	1.00	1.00
Angular	0.580	0.117	1.54	0.33	1.00	1.00
Angular	0.675	0.119	1.69	0.41	1.00	1.00
Angular	0.205	0.102	0.86	0.05	1.00	1.00
Rounded	0.444	0.124	1.09	0.09	1.36	1.00
Rounded	0.434	0.122	1.11	0.09	1.36	1.00
Angular	0.242	0.088	0.96	0.08	1.00	1.00
Angular	0.256	0.088	1.00	0.09	1.00	1.00
Angular	0.274	0.090	1.01	0.10	1.00	1.00
Rounded	0.396	0.092	1.31	0.19	1.36	1.00
Rounded	0.408	0.093	1.32	0.19	1.36	1.00
Angular	0.631	0.105	1.49	0.33	1.00	1.00
Angular	0.646	0.105	1.52	0.35	1.00	1.00
Rounded	0.521	0.122	0.92	0.06	1.36	1.00
Angular	0.672	0.105	1.55	0.38	1.00	1.05
Angular	0.182	0.055	0.99	0.42	1.00	1.00
Angular	0.063	0.044	0.62	0.10	1.00	1.00
Angular	0.106	0.042	0.82	0.14	1.00	1.00
Angular	0.093	0.069	0.71	0.03	1.00	1.00
Angular	0.093	0.070	0.69	0.03	1.00	1.00
Angular	0.086	0.068	0.69	0.03	1.00	1.00
Angular	0.100	0.070	0.74	0.04	1.00	1.00
Angular	0.236	0.084	1.05	0.10	1.00	1.00
Angular	0.252	0.085	1.07	0.12	1.00	1.00
Angular	0.252	0.085	1.07	0.12	1.00	1.00
Angular	0.279	0.081	1.16	0.17	1.00	1.00

[1] Data point not considered to be representative of the critical rock size and ultimately removed from the final best-fit equation analysis.

Table 20 – Adopted data parameters for the additional best-guess data points

d50	Slope	flow rate (failure)	Specific weight	d50/d90	Manning	Depth
(m)	(-)	(m³/s/m)	-	-	-	m
0.074	0.01	2.52	2.6	0.8	0.031	1.00
0.194	0.02	3.04	2.6	0.8	0.047	1.00
0.149	0.01	7.11	2.6	0.8	0.035	2.00
0.387	0.02	8.60	2.6	0.8	0.052	2.00
0.396	0.05	4.67	2.6	0.8	0.065	1.20
0.223	0.01	13.07	2.6	0.8	0.037	3.00
0.581	0.02	15.80	2.6	0.8	0.056	3.00
0.969	0.05	18.69	2.6	0.8	0.075	3.00
0.131	0.01	4.43	2.6	0.8	0.035	1.40
0.336	0.03	5.49	2.6	0.8	0.055	1.40
0.179	0.01	5.63	2.6	0.8	0.039	1.60
0.456	0.04	7.01	2.6	0.8	0.062	1.60
0.225	0.01	9.93	2.6	0.8	0.039	2.40
0.568	0.03	12.42	2.6	0.8	0.060	2.40
0.291	0.01	11.66	2.6	0.8	0.042	2.60
0.733	0.04	14.62	2.6	0.8	0.067	2.60
0.652	0.05	10.12	2.6	0.8	0.070	2.00

Table 21 – Adopted data parameters for the additional best-guess data points

Location	d90 (m)	Manning's roughness	Local velocity m/s)	Unit discharge	K1	K2
Theoretical	0.148	0.031	2.52	2.52	1.00	1.00
Theoretical	0.388	0.047	3.04	3.04	1.00	1.00
Theoretical	0.298	0.035	3.56	7.11	1.00	1.00
Theoretical	0.774	0.052	4.30	8.60	1.00	1.00
Theoretical	0.792	0.065	3.89	4.67	1.00	1.00
Theoretical	0.446	0.037	4.36	13.07	1.00	1.00
Theoretical	1.162	0.056	5.27	15.80	1.00	1.00
Theoretical	1.938	0.075	6.23	18.69	1.00	1.00
Theoretical	0.262	0.035	3.16	4.43	1.00	1.00
Theoretical	0.672	0.055	3.92	5.49	1.00	1.00
Theoretical	0.358	0.039	3.52	5.63	1.00	1.00
Theoretical	0.912	0.062	4.38	7.01	1.00	1.00
Theoretical	0.450	0.039	4.14	9.93	1.00	1.00
Theoretical	1.136	0.060	5.18	12.42	1.00	1.00
Theoretical	0.582	0.042	4.49	11.66	1.00	1.00
Theoretical	1.466	0.067	5.62	14.62	1.00	1.00
Theoretical	1.304	0.070	5.06	10.12	1.00	1.00

A best-fit analysis of the combined field, laboratory and best-guess deepwater data produced three equations. Equation 157 is a best-fit equation based on the same format as that presented by Abt, S.R. and Johnson, T.L. (1991).

$$\text{(uniform flow)} \quad d_{50,\text{critical}} = \frac{0.98(\text{SF}) \cdot K_1 \cdot K_2 \cdot S_o^{0.47} \cdot q^{0.64}}{(s_r - 1)} \quad (157)$$

Equation 158 represents the best-fit equation with the incorporation of a flow depth factor in line with the work of Maynard (1978, 1988) and Maynard, Ruff & Abt (1989).

$$\text{(uniform flow)} \quad d_{50,\text{critical}} = \frac{1.11(\text{SF}) \cdot K_1 \cdot K_2 \cdot S_o^{0.47} \cdot q^{0.48} \cdot y^{0.2}}{(s_r - 1)} \quad (158)$$

Equation 159 is a modification of Equation 158 that was found to produce slightly better fit for those flow conditions that would result in a large rock size, i.e. Equation 159 produced a better fit for the more critical high stress flow conditions.

$$\text{(uniform flow)} \quad d_{50,\text{critical}} = \frac{1.27(\text{SF}) \cdot K_1 \cdot K_2 \cdot S_o^{0.5} \cdot q^{0.5} \cdot y^{0.25}}{(s_r - 1)} \quad (159)$$

The critical (i.e. SF = 1.0) d_{50} rock sizes determined by Equations 157 to 159 in comparison with the observed values are presented in Tables 19 and 20.

After considering various issues and the primary purpose of this study, it was concluded that Equation 159 represented the best overall design equation.

Graphical representations of the output from Equation 159 in comparison with the observed critical rock size are presented in Figures 6 to 9.

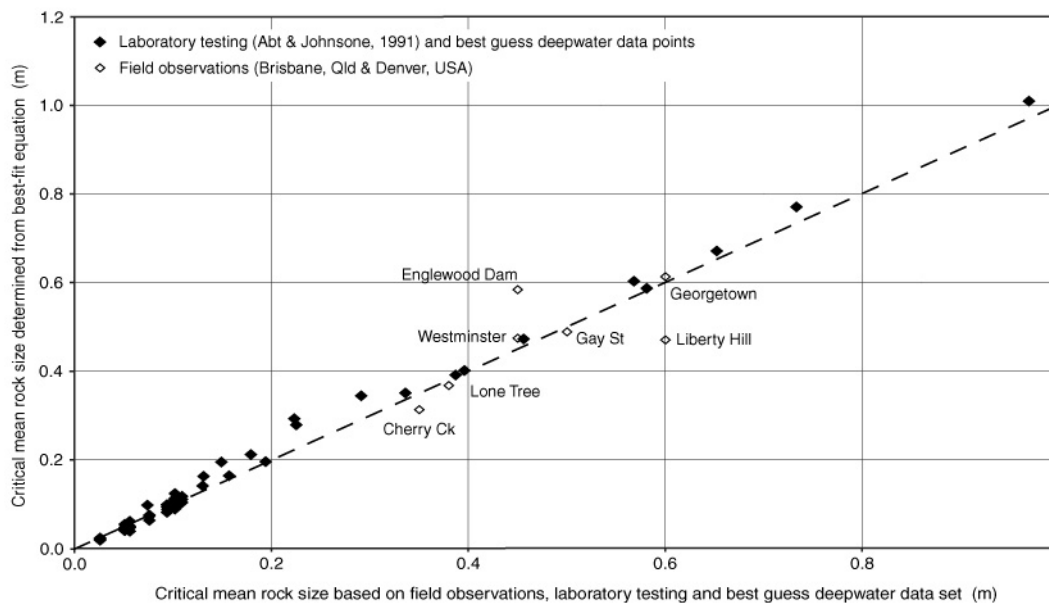


Figure 6 – Equation 3 output vs original data set (critical rock size, SF = 1.0)

Table 22 – Critical d_{50} rock size determined by the best-fit equations for the reduced field and laboratory data set

Location	Unit discharge	Flow depth (m)	Observed d_{50} (m)	Eqn 157 d_{50} (m)	Eqn 158 d_{50} (m)	Eqn 159 d_{50} (m)
Cherry Ck	1.49	0.53	350	317	297	313
Liberty Hill	1.96	0.65	600	464	434	471
Little Dry Ck	2.34	0.67	450	480	438	475
Lone Tree	2.74	0.81	380	370	343	368
Georgetown	3.02	0.79	600	612	555	613
Englewood	4.38	0.98	450	593	528	584
Gay St	5.00	1.29	500	480	443	489
Sandstone	0.12	0.07	76	86	82	75
Sandstone	0.08	0.06	76	84	80	73
Sandstone	0.05	0.04	76	72	70	63
Sandstone	0.23	0.11	109	125	116	110
Sandstone	0.16	0.09	109	117	110	103
Sandstone	0.16	0.08	109	134	124	117
Basalt	0.19	0.10	94	93	87	81
Basalt	0.18	0.09	94	109	99	94
Basalt	0.16	0.08	94	115	104	99
Angular	0.17	0.19	104	89	97	94
Angular	0.33	0.26	130	137	142	141
Angular	0.41	0.30	157	158	162	164
Angular	0.05	0.09	56	37	42	38
Rounded	0.09	0.17	102	80	93	89
Rounded	0.09	0.17	102	80	93	89
Angular	0.08	0.14	51	38	44	40
Angular	0.09	0.15	51	42	48	44
Angular	0.10	0.16	51	45	51	47
Rounded	0.19	0.24	102	102	114	109
Rounded	0.19	0.24	102	105	117	113
Angular	0.33	0.35	102	98	108	107
Angular	0.35	0.36	102	103	113	112
Rounded	0.06	0.21	51	45	58	54
Angular	0.38	0.37	102	114	124	124
Angular	0.42	0.24	56	52	51	47
Angular	0.10	0.09	26	21	21	18
Angular	0.14	0.20	26	18	21	18
Angular	0.03	0.06	26	22	24	21
Angular	0.03	0.06	26	21	24	21
Angular	0.03	0.06	26	20	23	19
Angular	0.04	0.07	26	24	27	23
Angular	0.10	0.15	56	45	50	47
Angular	0.12	0.15	56	49	53	50
Angular	0.12	0.15	56	49	53	50
Angular	0.17	0.19	56	60	65	61

Table 23 – Critical d_{50} rock size determined by the best-fit equations for the best-guess, deepwater data set

Location	Unit discharge	Flow depth (m)	Observed d_{50} (m)	Eqn 157 d_{50} (m)	Eqn 158 d_{50} (m)	Eqn 159 d_{50} (m)
Theoretical	2.52	1.00	74	100	98	98
Theoretical	3.04	1.00	194	198	188	196
Theoretical	7.11	2.00	149	194	185	195
Theoretical	8.60	2.00	387	386	356	392
Theoretical	4.67	1.20	396	401	369	401
Theoretical	13.07	3.00	223	286	268	293
Theoretical	15.80	3.00	581	569	517	587
Theoretical	18.69	3.00	969	975	862	1010
Theoretical	4.43	1.40	131	164	157	162
Theoretical	5.49	1.40	336	350	323	351
Theoretical	5.63	1.60	179	212	201	212
Theoretical	7.01	1.60	456	469	428	473
Theoretical	9.93	2.40	225	275	257	279
Theoretical	12.42	2.40	568	591	533	603
Theoretical	11.66	2.60	291	339	314	344
Theoretical	14.62	2.60	733	751	670	771
Theoretical	10.12	2.00	652	659	592	671

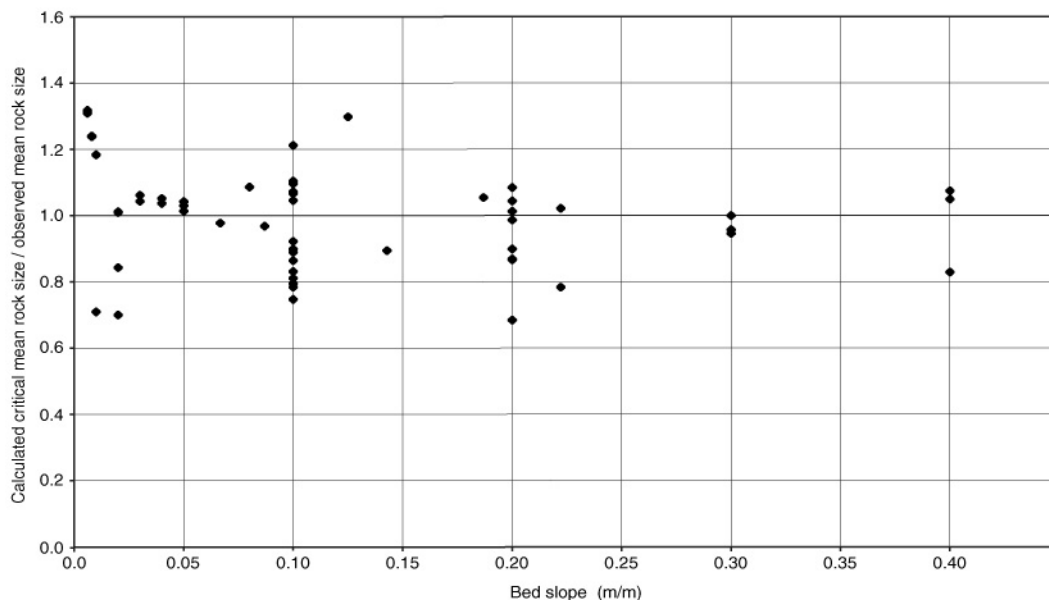


Figure 7 – Equation 3: ratio of output/observed vs bed slope, based on the critical rock size (SF = 1.0)

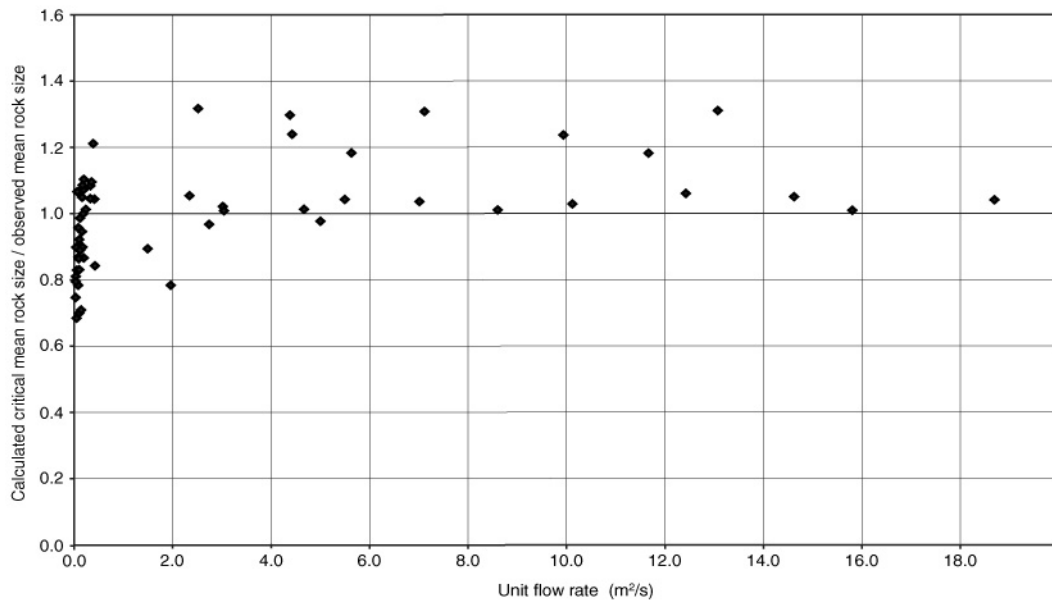


Figure 8 – Equation 3: ratio of output/observed vs unit flow rate, based on the critical rock size (SF = 1.0)

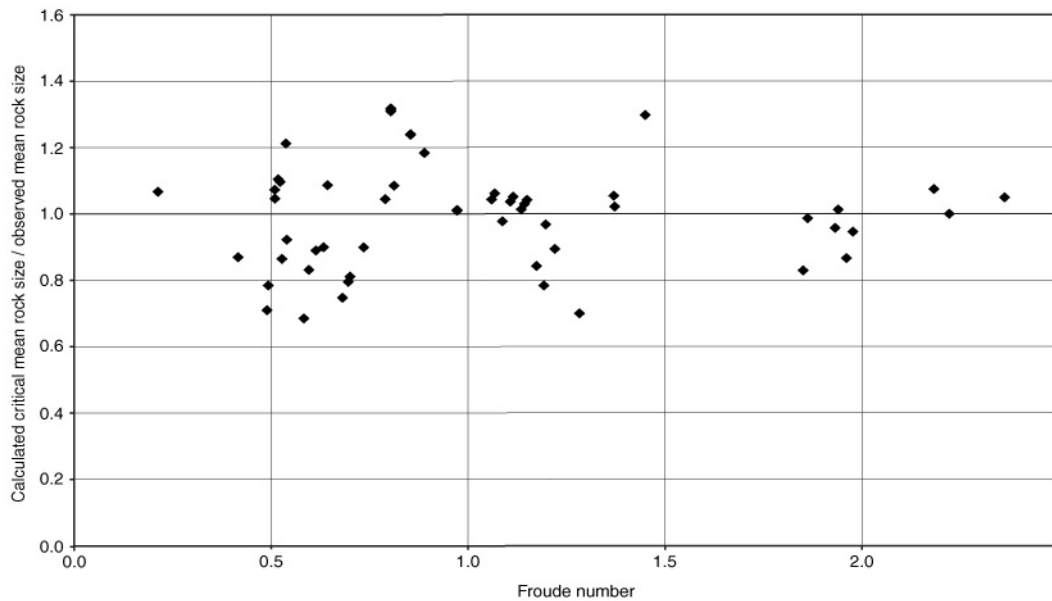


Figure 9 – Equation 3: ratio of output/observed vs Froude number, based on the critical rock size (SF = 1.0)

The results shown in Figures 7 to 9 indicate that a safety factor (SF) of 1.2 would in most cases provide a satisfactory design outcome resulting in a low risk of the design rock size being less than the actual critical rock size.

14.15 Development of various simplified design equations

To assist designers in the determination of a suitable rock size in various design circumstances, various simplified design equations were developed from Equation 159. These equations rely on different input data and each equation has been assessed to determine the applicable range of bed slopes.

A simplified, uniform flow equation that is independent of flow depth and suitable for bed slopes, $S_o < 50\%$.

$$d_{50} = \frac{SF \cdot K_1 \cdot K_2 \cdot S_o^{0.47} \cdot q^{0.64}}{(s_r - 1)} \quad (160)$$

A simplified, uniform flow equation based on velocity and bed slope, and suitable for bed slopes, $S_o < 33\%$.

$$d_{50} = \frac{SF \cdot K_1 \cdot K_2 \cdot V^2}{(A - B \cdot \ln(S_o)) \cdot (s_r - 1)} \quad (161)$$

For SF = 1.2: A = 3.95, B = 4.97

For SF = 1.5: A = 2.44, B = 4.60

A simplified, uniform flow equation based on velocity and flow depth, and suitable for bed slopes, $S_o < 10\%$.

$$d_{50} \approx \frac{K_1 \cdot V^{3.9}}{C \cdot y^{0.95} (s_r - 1)} \quad (162)$$

where: C = 120 and 68 for SF = 1.2 and 1.5 respectively

A simplified, non-uniform flow equation suitable for use in the design of partially drowned waterway chutes with a bed slope, $S_o < 50\%$.

$$d_{50} = \frac{1.27 \cdot SF \cdot K_1 \cdot K_2 \cdot S_o^{0.5} \cdot V^{2.5} \cdot y^{0.75}}{V_o^{2.0} (s_r - 1)} \quad (163)$$

A simplified, non-uniform flow, velocity-based equation suitable for use in non-critical conditions only, and where the bed slope, $S_o < 5\%$.

$$d_{50} \approx \frac{K_1 \cdot V^2}{2 \cdot g \cdot K^2 (s_r - 1)} \quad (164)$$

where: K = 1.10 for low turbulent, subcritical flow conditions, or 0.86 for high turbulent and/or supercritical flow conditions.

Equations 161 to 164 are based on Manning's "n" roughness being determined from Equation 11. Equation 164 represents a modification of the equation presented by Isbash (1936).

15.0 Recommendations

15.1 General issues

The recommended angles of repose of dumped rock are provided in Tables 1 and 2.

The recommended relative densities of various types of rock are provided in Table 3.

15.2 Rock sizing for channels and chutes

Table 24 provides the recommended design equations for sizing rock on the bed of channels and chutes. The same equations can be used for sizing rock on bank slopes equal to or less than 2:1, but a 25% increase in rock size should be applied for bank slopes of 1.5:1.

where:

d_{50} = nominal rock size (diameter) of which 50% of the rocks are smaller [m]

A & B = equation constants

K = equation constant based on flow conditions

= 1.10 for low turbulent, subcritical flow conditions, or 0.86 for high turbulent and/or supercritical flow conditions

K_1 = correction factor for rock shape

= 1.0 for angular (fractured) rock, 1.36 for rounded rock (i.e. smooth, spherical rock)

K_2 = correction factor for rock grading

= 0.95 for poorly graded rock ($C_u = d_{60}/d_{10} < 1.5$), 1.05 for well graded rock ($C_u > 2.5$), otherwise $K_2 = 1.0$ ($1.5 < C_u < 2.5$)

q = flow per unit width down the embankment [$m^3/s/m$]

s_r = specific gravity of rock

S_e = slope of energy line [m/m]

S_o = bed slope = $\tan(\theta)$ [m/m]

SF = factor of safety (refer to Table 2)

V = actual depth-average flow velocity at location of rock [m/s]

V_o = depth-average flow velocity based on **uniform** flow down a slope, S_o [m/s]

y = depth of flow at a given location [m]

θ = slope of channel bed [degrees]

Φ = angle of repose of rock [degrees]

Table 24 – Recommended rock sizing equations for channels and chutes

Bed slope (%)	Design equations
Preferred equation: $S_o < 50\%$	Uniform flow conditions only, $S_e = S_o$ $d_{50} = \frac{1.27 \cdot SF \cdot K_1 \cdot K_2 \cdot S_o^{0.5} \cdot q^{0.5} \cdot y^{0.25}}{(s_r - 1)} \quad (165)$
A simplified equation independent of flow depth: $S_o < 50\%$	Uniform flow conditions only, $S_e = S_o$ $d_{50} = \frac{SF \cdot K_1 \cdot K_2 \cdot S_o^{0.47} \cdot q^{0.64}}{(s_r - 1)} \quad (166)$
A simplified, velocity-based equation: $S_o < 33\%$	Uniform flow conditions only, $S_e = S_o$ $d_{50} = \frac{SF \cdot K_1 \cdot K_2 \cdot V^2}{(A - B \cdot \ln(S_o)) \cdot (s_r - 1)} \quad (167)$ For SF = 1.2: A = 3.95, B = 4.97 For SF = 1.5: A = 2.44, B = 4.60
A simplified, velocity-based equation: $S_o < 10\%$	Low gradient, uniform or non-uniform flow conditions $d_{50} \approx \frac{K_1 \cdot V^{3.9}}{C \cdot y^{0.95} (s_r - 1)} \quad (168)$ C = 120 and 68 for SF = 1.2 and 1.5 respectively
Partially drowned waterway chutes: $S_o < 50\%$	Steep gradient, non-uniform flow conditions, $S_e \neq S_o$ $d_{50} = \frac{1.27 \cdot SF \cdot K_1 \cdot K_2 \cdot S_o^{0.5} \cdot V^{2.5} \cdot y^{0.75}}{V_o^{2.0} (s_r - 1)} \quad (169)$
A simplified, velocity-based equation for non-critical conditions only: $S_o < 5\%$	Low gradient, non-uniform flow conditions $d_{50} \approx \frac{K_1 \cdot V^2}{2 \cdot g \cdot K^2 (s_r - 1)} \quad (170)$ K = 1.10 for low turbulent, subcritical flow conditions, or 0.86 for high turbulent and/or supercritical flow conditions

[1] The above equations, with the exception of Equation 166, are based on Manning's "n" roughness being determined from Equation 171.

Tables 26 to 32 provide design, mean rock size (rounded up to the next 0.1m unit) for a safety factor of 1.2 and 1.5, based on Equation 167 for $S_o = 0.1\%$ and Equation 165 for all other bed slopes.

Table 25 – Recommended safety factor for use in determining rock size

Safety factor (SF)	Recommended usage	Example site conditions
1.2	<ul style="list-style-type: none"> • Low risk structures. • Failure of structure is most unlikely to cause loss of life or irreversible property damage. • Permanent rock chutes with all voids filled with soil and pocket planted. 	<ul style="list-style-type: none"> • Embankment chutes where failure of the structure is likely to result in easily repairable soil erosion. • Permanent chutes that are likely to experience significant sedimentation and vegetation growth before experiencing the high flows. • Temporary (<2yrs) spillways with a design storm of 1 in 10 years of greater.
1.5	<ul style="list-style-type: none"> • High risk structures. • Failure of structure may cause loss of life or irreversible property damage. • Temporary structures that have a high risk of experiencing the design discharge while the voids remain open (i.e. prior to sediment settling within and stabilising the voids between individual rocks). 	<ul style="list-style-type: none"> • Waterway chutes where failure of the chute may cause severe gully erosion and/or damage to the waterway. • Sediment basin or dam spillways located immediately up-slope of a residential area or busy roadway where an embankment failure could cause property flooding or loss of life. • Spillways and chutes designed for a storm frequency less than 1 in 10 years.

Manning’s n roughness:

The roughness values presented in Table 1 have been developed from Equation 171. Equation 171 (Witheridge, 2002) was developed to allow estimation of the Manning's n of rock lined channels in shallow water.

$$n = \frac{d_{90}^{1/6}}{26(1 - 0.3593^{(X)^{0.7}})} \quad (171)$$

- where: X = (R/d₉₀)(d₅₀/d₉₀)
R = Hydraulic radius of flow over rocks [m]
d₅₀ = mean rock size for which 50% of rocks are smaller [m]
d₉₀ = mean rock size for which 90% of rocks are smaller [m]

For “natural” rock extracted from streambeds the relative roughness value (d₅₀/d₉₀) is typically in the range 0.2 to 0.5. For quarried rock the ratio is more likely to be in the range 0.5 to 0.8.

Table 26a – Preferred design table for mean rock size, d_{50} (m) for SF = 1.2

Safety factor, SF = 1.2		Specific gravity, $s_r = 2.4$				Size distribution, $d_{50}/d_{90} = 0.5$			
Unit flow rate ($m^3/s/m$)	Bed slope (%)								
	0.2	0.5	1.0	2.0	3.0	4.0	5.0	6.0	
0.2	0.05	0.05	0.05	0.05	0.10	0.10	0.10	0.10	
0.4	0.05	0.05	0.10	0.10	0.10	0.10	0.20	0.20	
0.6	0.05	0.05	0.10	0.10	0.20	0.20	0.20	0.20	
0.8	0.05	0.10	0.10	0.20	0.20	0.20	0.20	0.20	
1.0	0.05	0.10	0.10	0.20	0.20	0.20	0.30	0.30	
2.0	0.10	0.20	0.20	0.30	0.30	0.30	0.40	0.40	
3.0	0.10	0.20	0.20	0.30	0.40	0.40	0.50	0.50	
4.0	0.20	0.20	0.30	0.40	0.40	0.50	0.60	0.60	
5.0	0.20	0.30	0.30	0.40	0.50	0.60	0.60	0.70	
6.0	0.20	0.30	0.40	0.50	0.60	0.70	0.70	0.80	
7.0	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	
8.0	0.20	0.30	0.40	0.60	0.70	0.80	0.90	0.90	
9.0	0.20	0.30	0.50	0.60	0.70	0.80	0.90	1.00	
10.0	0.30	0.40	0.50	0.70	0.80	0.90	1.00	1.10	
15.0	0.30	0.50	0.60	0.80	1.00	1.20	1.30	1.40	
20.0	0.40	0.60	0.70	1.00	1.20	1.40	1.50	1.70	

Table 26b – Preferred design table for mean rock size, d_{50} (m) for SF = 1.2

Safety factor, SF = 1.2		Specific gravity, $s_r = 2.4$				Size distribution, $d_{50}/d_{90} = 0.5$			
Unit flow rate ($m^3/s/m$)	Bed slope (%)								
	8.0	9.0	10.0	15.0	20.0	25.0	33.3	50.0	
0.2	0.10	0.10	0.10	0.20	0.20	0.20	0.20	0.30	
0.4	0.20	0.20	0.20	0.20	0.30	0.30	0.30	0.40	
0.6	0.20	0.20	0.30	0.30	0.30	0.40	0.40	0.50	
0.8	0.30	0.30	0.30	0.30	0.40	0.40	0.50	0.60	
1.0	0.30	0.30	0.30	0.40	0.40	0.50	0.60	0.70	
2.0	0.50	0.50	0.50	0.60	0.70	0.70	0.90	1.00	
3.0	0.60	0.60	0.60	0.80	0.90	1.00	1.10	1.30	
4.0	0.70	0.70	0.80	0.90	1.00	1.20	1.30	1.60	
5.0	0.80	0.80	0.90	1.10	1.20	1.30	1.50	1.80	
6.0	0.90	0.90	1.00	1.20	1.40	1.50	1.70	2.10	
7.0	1.00	1.00	1.10	1.30	1.50	1.70	1.90		
8.0	1.10	1.10	1.20	1.40	1.60	1.80	2.10		
9.0	1.10	1.20	1.30	1.50	1.80	1.90	2.20		
10.0	1.20	1.30	1.40	1.60	1.90	2.10			
15.0	1.60	1.70	1.80	2.10	2.40				
20.0	1.90	2.00	2.10						

Table 27a – Alternative design table for mean rock size, d_{50} (m) for SF = 1.2

Safety factor, SF = 1.2		Specific gravity, $s_r = 2.4$				Size distribution, $d_{50}/d_{90} = 0.5$		
Local velocity (m/s)	Bed slope (%)							
	0.1	0.5	1.0	2.0	3.0	4.0	5.0	6.0
0.5	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
0.8	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
1.0	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
1.3	0.05	0.05	0.05	0.10	0.10	0.10	0.10	0.10
1.5	0.10	0.10	0.10	0.10	0.10	0.10	0.20	0.20
1.8	0.10	0.10	0.10	0.20	0.20	0.20	0.20	0.20
2.0	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20
2.3	0.20	0.20	0.20	0.20	0.30	0.30	0.30	0.30
2.5	0.20	0.20	0.20	0.30	0.30	0.30	0.30	0.40
2.8	0.20	0.30	0.30	0.30	0.40	0.40	0.40	0.40
3.0	0.30	0.30	0.30	0.40	0.40	0.40	0.50	0.50
3.5	0.40	0.40	0.40	0.50	0.50	0.60	0.60	0.70
4.0	0.50	0.50	0.50	0.60	0.70	0.80	0.80	0.80
4.5	0.60	0.60	0.70	0.80	0.90	0.90	1.00	1.10
5.0	0.70	0.70	0.80	1.00	1.10	1.10	1.20	1.30
6.0	1.00	1.00	1.20	1.40	1.50	1.60	1.70	1.80

Table 27b – Alternative design table for mean rock size, d_{50} (m) for SF = 1.2

Safety factor, SF = 1.2		Specific gravity, $s_r = 2.4$				Size distribution, $d_{50}/d_{90} = 0.5$		
Local velocity (m/s)	Bed slope (%)							
	8.0	9.0	10.0	15.0	20.0	25.0	33.3	50.0
0.5	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
0.8	0.05	0.05	0.05	0.05	0.05	0.05	0.10	0.10
1.0	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.20
1.3	0.10	0.10	0.10	0.20	0.20	0.20	0.20	0.20
1.5	0.20	0.20	0.20	0.20	0.20	0.20	0.30	0.30
1.8	0.20	0.20	0.20	0.30	0.30	0.30	0.30	0.40
2.0	0.30	0.30	0.30	0.30	0.30	0.40	0.40	0.50
2.3	0.30	0.30	0.30	0.40	0.40	0.50	0.50	0.60
2.5	0.40	0.40	0.40	0.50	0.50	0.60	0.60	0.70
2.8	0.50	0.50	0.50	0.60	0.60	0.70	0.70	0.80
3.0	0.50	0.60	0.60	0.70	0.70	0.80	0.90	1.00
3.5	0.70	0.70	0.80	0.90	1.00	1.00	1.10	1.30
4.0	0.90	1.00	1.00	1.10	1.20	1.30	1.50	1.70
4.5	1.10	1.20	1.20	1.40	1.50	1.70	1.90	2.10
5.0	1.40	1.50	1.50	1.70	1.90	2.00	2.20	
6.0	1.90	2.00	2.10	2.40	2.70			

Table 28a – Alternative design table for mean rock size, d_{50} (m) for SF = 1.2

Safety factor, SF = 1.2		Specific gravity, $s_r = 2.4$				Size distribution, $d_{50}/d_{90} = 0.5$			
Local flow depth (m)	Bed slope (%)								
	0.1	0.5	1.0	2.0	3.0	4.0	5.0	6.0	
0.1	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	
0.2	0.05	0.05	0.05	0.10	0.10	0.10	0.10	0.10	
0.4	0.05	0.05	0.10	0.20	0.20	0.20	0.20	0.20	
0.6	0.05	0.10	0.10	0.20	0.20	0.30	0.30	0.30	
0.8	0.05	0.10	0.20	0.30	0.30	0.30	0.40	0.40	
1.0	0.10	0.20	0.20	0.30	0.40	0.40	0.50	0.50	
1.2	0.10	0.20	0.20	0.40	0.40	0.50	0.60	0.60	
1.4	0.10	0.20	0.30	0.40	0.50	0.60	0.60	0.70	
1.6	0.10	0.20	0.30	0.50	0.60	0.60	0.70	0.80	
1.8	0.20	0.20	0.30	0.50	0.60	0.70	0.80	0.90	
2.0	0.20	0.30	0.40	0.60	0.70	0.80	0.90	1.00	
2.2	0.20	0.30	0.40	0.60	0.70	0.90	1.00	1.10	
2.4	0.20	0.30	0.40	0.70	0.80	0.90	1.10	1.20	
2.6	0.20	0.30	0.50	0.70	0.90	1.00	1.20	1.30	
2.8	0.20	0.40	0.50	0.80	0.90	1.10	1.20	1.40	
3.0	0.20	0.40	0.50	0.80	1.00	1.20	1.30	1.50	

Table 28b – Alternative design table for mean rock size, d_{50} (m) for SF = 1.2

Safety factor, SF = 1.2		Specific gravity, $s_r = 2.4$				Size distribution, $d_{50}/d_{90} = 0.5$			
Local flow depth (m)	Bed slope (%)								
	8.0	9.0	10.0	15.0	20.0	25.0	33.3	50.0	
0.1	0.10	0.10	0.10	0.10	0.10	0.20	0.20	0.20	
0.2	0.20	0.20	0.20	0.20	0.20	0.30	0.30	0.40	
0.4	0.30	0.30	0.30	0.40	0.40	0.50	0.50	0.70	
0.6	0.40	0.40	0.40	0.50	0.60	0.70	0.80	1.00	
0.8	0.50	0.50	0.60	0.70	0.80	0.90	1.00	1.30	
1.0	0.60	0.60	0.70	0.80	1.00	1.10	1.30	1.60	
1.2	0.70	0.80	0.80	1.00	1.20	1.30	1.50	1.90	
1.4	0.80	0.90	0.90	1.10	1.30	1.50	1.70	2.20	
1.6	0.90	1.00	1.10	1.30	1.50	1.70	2.00		
1.8	1.00	1.10	1.20	1.50	1.70	1.90	2.20		
2.0	1.20	1.20	1.30	1.60	1.90	2.10			
2.2	1.30	1.30	1.40	1.80	2.10				
2.4	1.40	1.50	1.60	1.90	2.30				
2.6	1.50	1.60	1.70	2.10					
2.8	1.60	1.70	1.80	2.20					
3.0	1.70	1.80	1.90	2.40					

Table 29a – Preferred design table for mean rock size, d_{50} (m) for SF = 1.5

Safety factor, SF = 1.5		Specific gravity, $s_r = 2.4$			Size distribution, $d_{50}/d_{90} = 0.5$			
Unit flow rate ($m^3/s/m$)	Bed slope (%)							
	0.1	0.5	1.0	2.0	3.0	4.0	5.0	6.0
0.2	0.05	0.05	0.05	0.10	0.10	0.10	0.10	0.10
0.4	0.05	0.05	0.10	0.10	0.20	0.20	0.20	0.20
0.6	0.05	0.10	0.10	0.20	0.20	0.20	0.20	0.30
0.8	0.05	0.10	0.20	0.20	0.20	0.20	0.30	0.30
1.0	0.10	0.10	0.20	0.20	0.30	0.30	0.30	0.30
2.0	0.10	0.20	0.20	0.30	0.40	0.40	0.50	0.50
3.0	0.20	0.20	0.30	0.40	0.50	0.50	0.60	0.60
4.0	0.20	0.30	0.40	0.50	0.60	0.60	0.70	0.80
5.0	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
6.0	0.20	0.30	0.40	0.60	0.70	0.80	0.90	1.00
7.0	0.30	0.40	0.50	0.70	0.80	0.90	1.00	1.10
8.0	0.30	0.40	0.50	0.70	0.90	1.00	1.10	1.20
9.0	0.30	0.40	0.60	0.80	0.90	1.10	1.20	1.30
10.0	0.30	0.50	0.60	0.80	1.00	1.10	1.20	1.40
15.0	0.40	0.60	0.80	1.10	1.30	1.50	1.60	1.80
20.0	0.50	0.70	0.90	1.30	1.50	1.80	1.90	2.10

Table 29b – Preferred design table for mean rock size, d_{50} (m) for SF = 1.5

Safety factor, SF = 1.5		Specific gravity, $s_r = 2.4$			Size distribution, $d_{50}/d_{90} = 0.5$			
Unit flow rate ($m^3/s/m$)	Bed slope (%)							
	8.0	9.0	10.0	15.0	20.0	25.0	33.3	50.0
0.2	0.20	0.20	0.20	0.20	0.20	0.20	0.30	0.30
0.4	0.20	0.20	0.20	0.30	0.30	0.40	0.40	0.50
0.6	0.30	0.30	0.30	0.40	0.40	0.40	0.50	0.60
0.8	0.30	0.30	0.40	0.40	0.50	0.50	0.60	0.70
1.0	0.40	0.40	0.40	0.50	0.60	0.60	0.70	0.80
2.0	0.60	0.60	0.60	0.70	0.80	0.90	1.10	1.30
3.0	0.70	0.80	0.80	1.00	1.10	1.20	1.40	1.70
4.0	0.90	0.90	1.00	1.20	1.30	1.50	1.70	2.00
5.0	1.00	1.00	1.10	1.30	1.50	1.70	1.90	2.30
6.0	1.10	1.20	1.20	1.50	1.70	1.90	2.20	
7.0	1.20	1.30	1.40	1.70	1.90	2.10		
8.0	1.30	1.40	1.50	1.80	2.10			
9.0	1.40	1.50	1.60	1.90	2.20			
10.0	1.50	1.60	1.70	2.10				
15.0	2.00	2.10	2.20					
20.0	2.40							

Table 30a – Alternative design table for mean rock size, d_{50} (m) for SF = 1.5

Safety factor, SF = 1.5		Specific gravity, $s_r = 2.4$				Size distribution, $d_{50}/d_{90} = 0.5$		
Local velocity (m/s)	Bed slope (%)							
	0.1	0.5	1.0	2.0	3.0	4.0	5.0	6.0
0.5	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
0.8	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
1.0	0.05	0.05	0.05	0.10	0.10	0.10	0.10	0.10
1.3	0.10	0.10	0.10	0.10	0.10	0.10	0.20	0.20
1.5	0.10	0.10	0.10	0.20	0.20	0.20	0.20	0.20
1.8	0.20	0.20	0.20	0.20	0.20	0.20	0.30	0.30
2.0	0.20	0.20	0.20	0.30	0.30	0.30	0.30	0.30
2.3	0.20	0.20	0.30	0.30	0.30	0.40	0.40	0.40
2.5	0.30	0.30	0.30	0.40	0.40	0.40	0.50	0.50
2.8	0.30	0.30	0.40	0.40	0.50	0.50	0.60	0.60
3.0	0.40	0.40	0.40	0.50	0.60	0.60	0.70	0.70
3.5	0.50	0.50	0.60	0.70	0.80	0.80	0.90	0.90
4.0	0.60	0.70	0.80	0.90	1.00	1.10	1.10	1.20
4.5	0.80	0.80	0.90	1.10	1.20	1.30	1.40	1.50
5.0	0.90	1.00	1.10	1.40	1.50	1.60	1.80	1.90
6.0	1.30	1.40	1.60	1.90	2.10	2.30	2.50	2.60

Table 30b – Alternative design table for mean rock size, d_{50} (m) for SF = 1.5

Safety factor, SF = 1.5		Specific gravity, $s_r = 2.4$				Size distribution, $d_{50}/d_{90} = 0.5$		
Local velocity (m/s)	Bed slope (%)							
	8.0	9.0	10.0	15.0	20.0	25.0	33.3	50.0
0.5	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
0.8	0.05	0.05	0.05	0.10	0.10	0.10	0.10	0.10
1.0	0.10	0.10	0.10	0.10	0.20	0.20	0.20	0.20
1.3	0.20	0.20	0.20	0.20	0.20	0.20	0.30	0.30
1.5	0.20	0.20	0.20	0.30	0.30	0.30	0.30	0.40
1.8	0.30	0.30	0.30	0.40	0.40	0.40	0.50	0.50
2.0	0.40	0.40	0.40	0.40	0.50	0.50	0.60	0.70
2.3	0.50	0.50	0.50	0.50	0.60	0.60	0.70	0.80
2.5	0.50	0.60	0.60	0.70	0.70	0.80	0.90	1.00
2.8	0.60	0.70	0.70	0.80	0.90	0.90	1.00	1.20
3.0	0.80	0.80	0.80	0.90	1.00	1.10	1.20	1.40
3.5	1.00	1.10	1.10	1.30	1.40	1.50	1.70	1.90
4.0	1.30	1.40	1.40	1.60	1.80	1.90	2.10	2.50
4.5	1.70	1.70	1.80	2.00	2.20	2.40		
5.0	2.00	2.10	2.10	2.50				
6.0	2.80							

Table 31a – Alternative design table for mean rock size, d_{50} (m) for SF = 1.5

Safety factor, SF = 1.5		Specific gravity, $s_r = 2.4$				Size distribution, $d_{50}/d_{90} = 0.5$		
Local flow depth (m)	Bed slope (%)							
	0.1	0.5	1.0	2.0	3.0	4.0	5.0	6.0
0.1	0.05	0.05	0.05	0.05	0.05	0.05	0.10	0.10
0.2	0.05	0.05	0.05	0.10	0.10	0.10	0.20	0.20
0.4	0.05	0.10	0.10	0.20	0.20	0.20	0.30	0.30
0.6	0.05	0.10	0.20	0.20	0.30	0.30	0.40	0.40
0.8	0.10	0.20	0.20	0.30	0.40	0.40	0.50	0.50
1.0	0.10	0.20	0.20	0.30	0.40	0.50	0.60	0.60
1.2	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.70
1.4	0.20	0.20	0.30	0.50	0.60	0.70	0.80	0.80
1.6	0.20	0.30	0.40	0.50	0.70	0.80	0.90	0.90
1.8	0.20	0.30	0.40	0.60	0.70	0.80	1.00	1.00
2.0	0.20	0.30	0.40	0.60	0.80	0.90	1.10	1.20
2.2	0.20	0.30	0.50	0.70	0.90	1.00	1.20	1.30
2.4	0.20	0.40	0.50	0.80	1.00	1.10	1.30	1.40
2.6	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.50
2.8	0.30	0.40	0.60	0.90	1.10	1.30	1.50	1.60
3.0	0.30	0.40	0.60	0.90	1.20	1.40	1.60	1.70

Table 31b – Alternative design table for mean rock size, d_{50} (m) for SF = 1.5

Safety factor, SF = 1.5		Specific gravity, $s_r = 2.4$				Size distribution, $d_{50}/d_{90} = 0.5$		
Local flow depth (m)	Bed slope (%)							
	8.0	9.0	10.0	15.0	20.0	25.0	33.3	50.0
0.1	0.10	0.10	0.10	0.10	0.20	0.20	0.20	0.20
0.2	0.20	0.20	0.20	0.20	0.30	0.30	0.30	0.40
0.4	0.30	0.30	0.30	0.40	0.50	0.50	0.60	0.80
0.6	0.40	0.50	0.50	0.60	0.70	0.80	0.90	1.10
0.8	0.60	0.60	0.60	0.80	0.90	1.00	1.20	1.50
1.0	0.70	0.70	0.80	1.00	1.10	1.30	1.50	1.80
1.2	0.80	0.90	0.90	1.20	1.30	1.50	1.80	2.20
1.4	1.00	1.00	1.10	1.30	1.60	1.80	2.00	
1.6	1.10	1.20	1.20	1.50	1.80	2.00		
1.8	1.20	1.30	1.40	1.70	2.00			
2.0	1.40	1.40	1.50	1.90	2.20			
2.2	1.50	1.60	1.70	2.10				
2.4	1.60	1.70	1.80	2.30				
2.6	1.70	1.90	2.00					
2.8	1.90	2.00						
3.0	2.00							

Table 32a – Alternative design table for mean rock size for specific gravity, $s_r = 2.6$

Safety factor, SF = 1.5		Specific gravity, $s_r = 2.6$				Size distribution, $d_{50}/d_{90} = 0.5$		
Local flow depth (m)	Bed slope (%)							
	0.1	0.5	1.0	2.0	3.0	4.0	5.0	6.0
0.1	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.10
0.2	0.05	0.05	0.05	0.10	0.10	0.10	0.10	0.20
0.4	0.05	0.05	0.10	0.20	0.20	0.20	0.20	0.30
0.6	0.05	0.10	0.20	0.20	0.30	0.30	0.30	0.40
0.8	0.10	0.10	0.20	0.30	0.30	0.40	0.40	0.50
1.0	0.10	0.20	0.20	0.30	0.40	0.40	0.50	0.60
1.2	0.10	0.20	0.30	0.40	0.50	0.50	0.60	0.70
1.4	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80
1.6	0.20	0.20	0.30	0.50	0.60	0.70	0.80	0.90
1.8	0.20	0.30	0.40	0.50	0.70	0.80	0.90	1.00
2.0	0.20	0.30	0.40	0.60	0.70	0.80	1.00	1.10
2.2	0.20	0.30	0.40	0.60	0.80	0.90	1.00	1.20
2.4	0.20	0.30	0.50	0.70	0.90	1.00	1.10	1.30
2.6	0.20	0.40	0.50	0.70	0.90	1.10	1.20	1.40
2.8	0.20	0.40	0.50	0.80	1.00	1.20	1.30	1.50
3.0	0.20	0.40	0.60	0.90	1.10	1.20	1.40	1.60

Table 32b – Alternative design table for mean rock size for specific gravity, $s_r = 2.6$

Safety factor, SF = 1.5		Specific gravity, $s_r = 2.6$				Size distribution, $d_{50}/d_{90} = 0.5$		
Local flow depth (m)	Bed slope (%)							
	8.0	9.0	10.0	15.0	20.0	25.0	33.3	50.0
0.1	0.10	0.10	0.10	0.10	0.10	0.20	0.20	0.20
0.2	0.20	0.20	0.20	0.20	0.20	0.30	0.30	0.40
0.4	0.30	0.30	0.30	0.40	0.40	0.50	0.60	0.70
0.6	0.40	0.40	0.50	0.60	0.60	0.70	0.80	1.00
0.8	0.50	0.60	0.60	0.70	0.80	0.90	1.10	1.30
1.0	0.60	0.70	0.70	0.90	1.00	1.20	1.30	1.70
1.2	0.80	0.80	0.90	1.10	1.20	1.40	1.60	2.00
1.4	0.90	0.90	1.00	1.20	1.40	1.60	1.90	2.30
1.6	1.00	1.10	1.10	1.40	1.60	1.80	2.10	
1.8	1.10	1.20	1.30	1.60	1.80	2.00		
2.0	1.20	1.30	1.40	1.70	2.00			
2.2	1.30	1.40	1.50	1.90	2.20			
2.4	1.50	1.60	1.70	2.10				
2.6	1.60	1.70	1.80	2.20				
2.8	1.70	1.80	1.90	2.40				
3.0	1.80	1.90	2.10					

15.3 Rock sizing on the outside bank of channel bends

Rock placed on the outside bank of **major** drainage and waterway channels may be sized using Equation 165 provided the bank slope does not exceed 2:1. A 25% increase in rock size should be applied for bank slopes of 1.5:1. The terms used in Equation 165 are defined below:

V = local flow velocity adjacent bank determined from Equations 4 or 5 as appropriate.

y = depth of flow measured to the toe of the bank.

S_o = slope of main channel bed (not the slope of the bank).

For constructed trapezoidal channel bends, the local flow velocity should be taken as 4/3 the average channel velocity, i.e.

$$V_{\text{bend}} = (4/3) V_{\text{avg}} \quad (172)$$

For natural channels, the local flow velocity should be taken as 1.5 times the average channel velocity:

$$V_{\text{bend}} = 1.5 V_{\text{avg}} \quad (173)$$

15.4 Rock sizing downstream of energy dissipation basins

Equation 174 is the recommended equation for sizing rock placed within the zone of highly turbulent water immediately **downstream** of the end sill of an **energy dissipater** (i.e. not within the main energy dissipation zone).

$$d_{50} = \frac{(0.081) \cdot V^{2.23}}{(s_r - 1)} \quad (174)$$

where: V = local, depth-average flow velocity immediately downstream of the end sill [m/s]

Rock sizing for other types of energy dissipaters, such as plunge pool systems, should be based on design guidelines specifically developed for the given type of energy dissipater.

15.5 Rock sizing of outlet pads for pipe outlets

The recommended equations for sizing rocks used within outlet protection immediately downstream of pipe outlets are presented below. These equations are based on the work of Bohan (1970).

Low tailwater ($TW < D_o/2$): $d_{50} = 0.25 D_o F_o$ (175)

which can be converted to: $d_{50} = 0.080 D_o^{1/2} V_o$ (176)

High tailwater ($TW > D_o/2$): $d_{50} = 0.25 D_o F_o - 0.15 D_o$ (177)

which can be converted to: $d_{50} = 0.080 D_o^{1/2} V_o - 0.15 D_o$ (178)

where:

TW = Tailwater depth relative to pipe invert [m]

D_o = internal diameter of outlet pipe [m]

F_o = Froude number of flow at the pipe exit [dimensionless]

16.0 Symbols and terminology

- a = effective shear stress surface area of rock [m²]
- A = cross sectional area of channel [m²]
- A = equation constant that typically varies with the degree of turbulence, rock texture (e.g. rounded or fractured) and the required factor of safety
- B = constant that typically varies with the degree of turbulence
- C_u = coefficient of uniformity in relation to the distribution of rock size = d₆₀/d₁₀
- d = nominal rock diameter [m]
- d_x = nominal rock size (diameter) of which X% of the rocks are smaller [m]
- d_{x,critical} = critical rock diameter for which movement is just expected [m]
- d_{50,critical} = mean rock diameter for which movement is expected to commence [m]
- D_o = internal diameter of outlet pipe [m]
- F = resultant force on rock [N]
- F = Froude number based on flow depth (y) rather than hydraulic radius [dimensionless]
- F* = Shield's parameter (variable depending on Reynolds number and rock size, d_x)
- F_d = drag force of rock [N]
- F_o = Froude number of flow at the pipe exit [dimensionless]
- F_R = resisting force on rock [N]
- F_w = weight component of water cell parallel to channel bed [N]
- g = acceleration due to gravity [m/s²]
- k = roughness coefficient (Strickler's coefficient)
- K = equation constant based on flow conditions (degree of local turbulence)
- K₁ = correction factor for rock shape
- K₂ = correction factor for rock grading
- L = length of water cell or control volume [m]
- n = Manning's roughness value [dimensionless]
- n_o = Manning's roughness value for deepwater conditions [dimensionless]
- P = wetted perimeter [m]
- q = flow per unit width down the embankment [m³/s/m]
- Q = total discharge [m³/s]
- R = hydraulic radius (R = A/P) [m]
- s_r = specific gravity of rock
- S = slope [m/m]
- S_e = slope of energy line [m/m]
- S_o = bed slope [m/m]
- SF = factor of safety as applied to rock size (d)
- SF_F = factor of safety as applied to the net displacement force of the rock (F)
 - = the ratio of the moments of forces resisting rotation of the rock particle out of the riprap blanket to the moments tending to dislodge the particle out of the riprap layer into the flow
- SF_V = factor of safety as applied to the design flow velocity (V)
- SF_W = factor of safety as applied to rock weight (W)
- TW = Tailwater depth relative to pipe invert [m]
- V = average flow velocity [m/s]
 - = local flow velocity average over depth [m/s]

- V_{avg} = the average channel velocity
- V_{bend} = the maximum depth-average velocity in the bend over the toe of the side slope
- W = weight of water cell [N]
- W = weight of d_{33} rock of which 67% of rock is larger [kg]
- y = depth of flow to bed at a given location [m]
- α = bank slope relative to the channel bed [degrees]
- β = bank slope relative to the horizontal [degrees]
- θ = slope of channel bed [degrees]
- ρ_w = density of water [kg/m^3]
- τ_c = critical shear stress at which point rocks begin to move [N/m^2]
- τ_o = average shear stress on the surface (bed and banks) of the channel [$\text{Pa} = \text{N}/\text{m}^2$]
- Φ = angle of repose of rock [degrees]
- φ = local gradient of energy line [degrees]

17.0 References

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